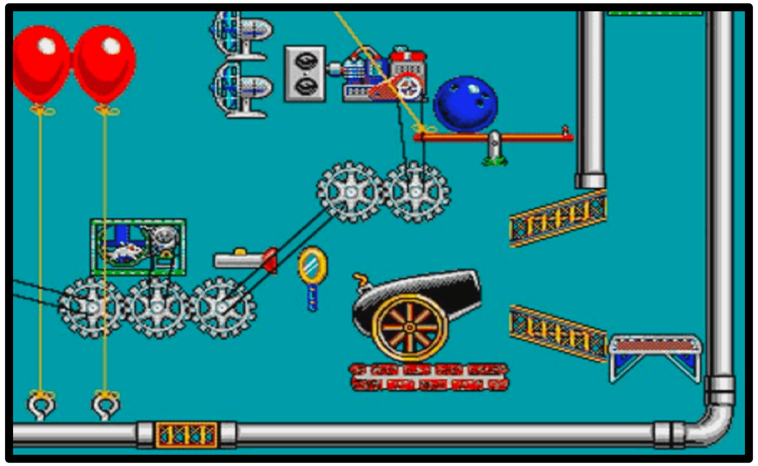
Bottom-up Synthesis of Memory Mutations with Separation Logic

Kasra Ferdowsi, Hila Peleg

Programming by Example

$\text{input}_1 \rightarrow \text{output}_1$
$\text{input}_2 \rightarrow \text{output}_2$
$\text{input}_3 \rightarrow \text{output}_3$

$\Downarrow$



$\Downarrow$

p

Programming by Example

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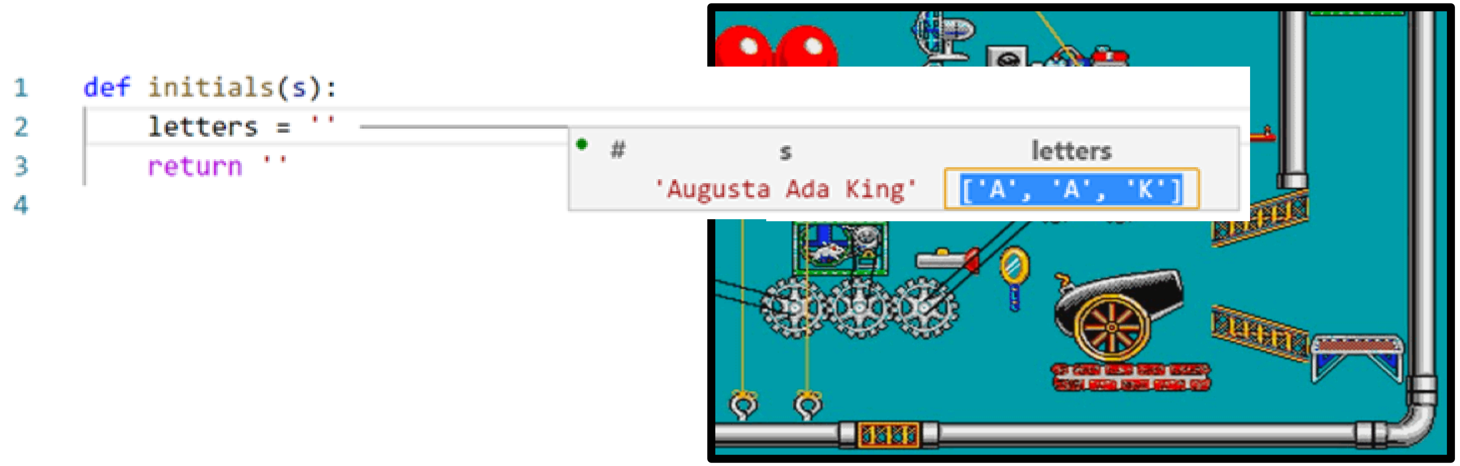
$$\Downarrow$$



$$\Downarrow$$
$$p$$

Programming by Example

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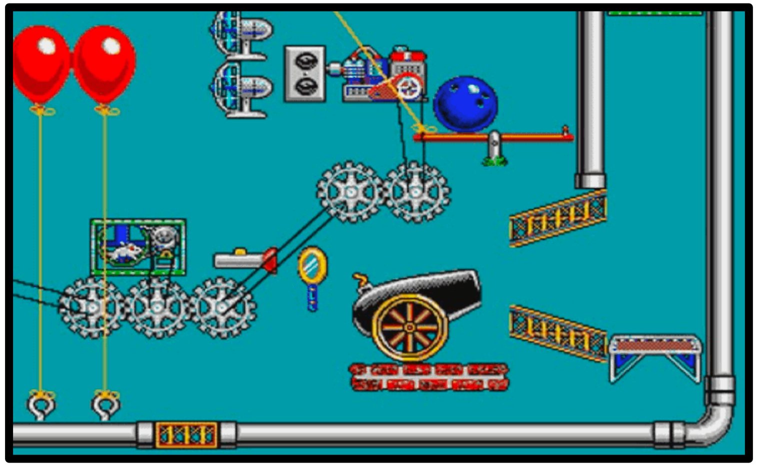


$\Downarrow$

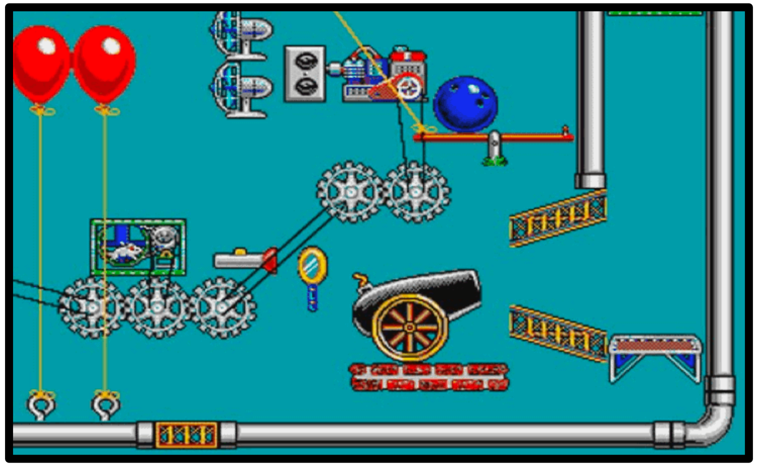
$$p$$

Counterexample
(input → output)

Programming by Example

Task: sum the min and max values in array of length n

⇓



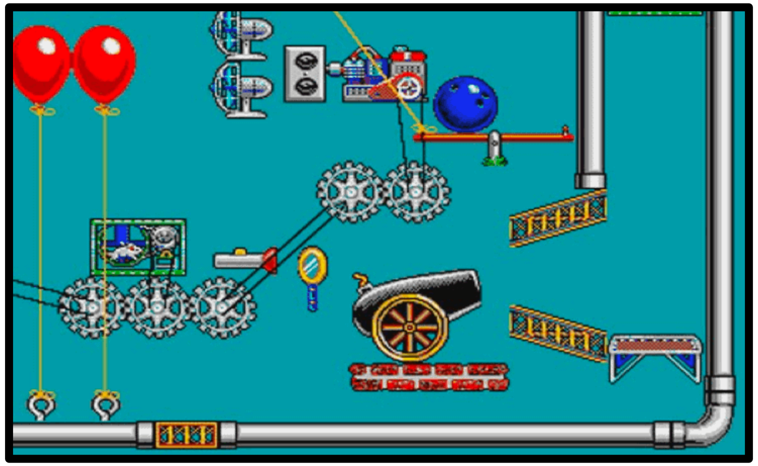
⇓

Programming by Example

Task: sum the min and max values in array of length n

{arr ↦[10,100,90,−1,2], n↦5} → 99

⇓



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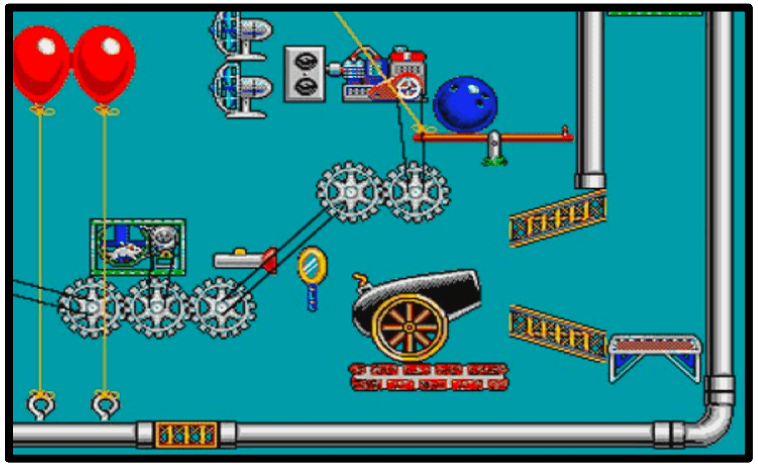
Programming by Example

Task: sum the min and max values in array of length n

$\{arr \mapsto [10,100,90,-1,2],\ n \mapsto 5\} \rightarrow 99$

$\Downarrow$



$\Downarrow$

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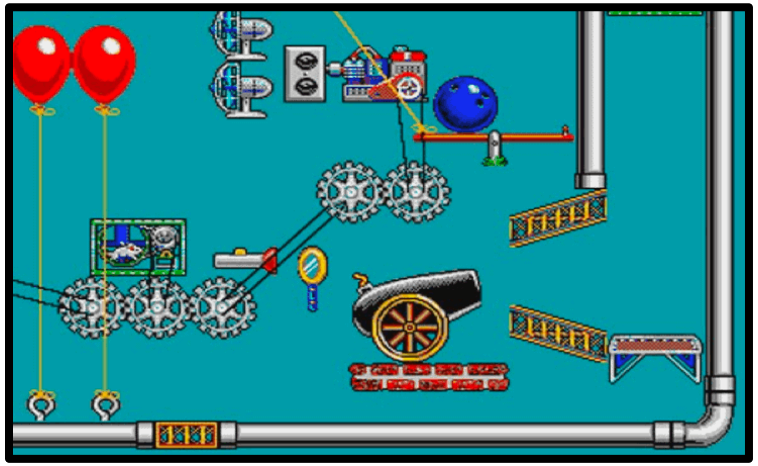

⇓

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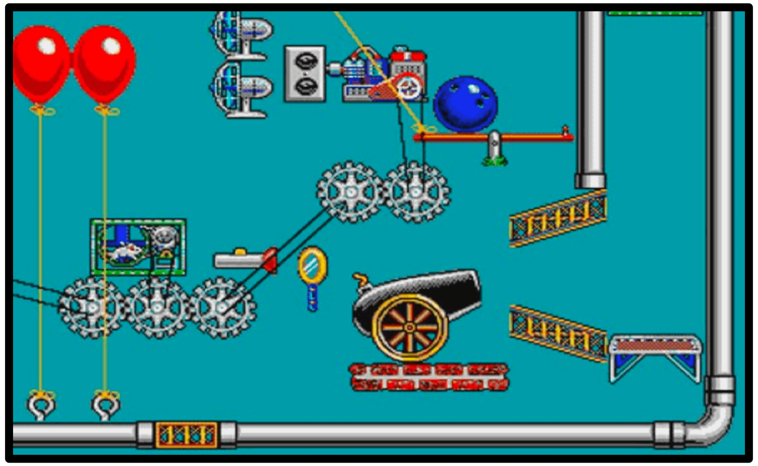

$\Downarrow$

```
arr.sort()[0] + arr[n-1]
```

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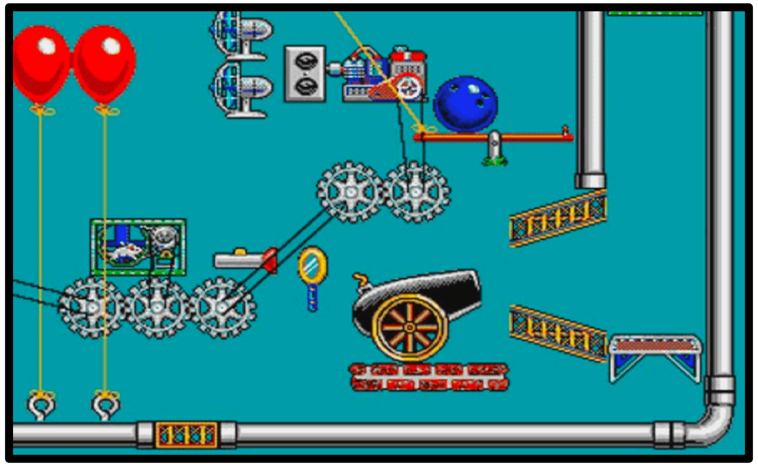

⇓

```
arr.sort()[0] + arr[n-1]
```

in-place 😱

Observational Equivalence

Enumerate space and test:

`n, arr, 0, 1, n + 0 n - 0, n + 1, n - 1, arr[0],…`

Observational Equivalence

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`n, arr, 0, 1, n + 0 n - 0, n + 1, n - 1, arr[0],...`

Observational Equivalence

ESCHER: Albarghouthi et al. 2013
TRANSIT: Udupa et al. 2013

Inputs that matter:

$\iota_1 = \{\texttt{arr} \mapsto [10,100,90,-1,2],\ \texttt{n} \mapsto 5\}$

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n⟨5⟩

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$n\langle 5 \rangle$

eval

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n⟨5⟩ arr⟨[10,100,90,−1,2]⟩

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n⟨5⟩ arr⟨[10,100,90,−1,2]⟩ 0⟨0⟩

eval

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ι_1 = {arr ↦[10,100,90,−1,2], n↦5}

n⟨5⟩   arr⟨[10,100,90,−1,2]⟩   0⟨0⟩   1⟨1⟩

eval

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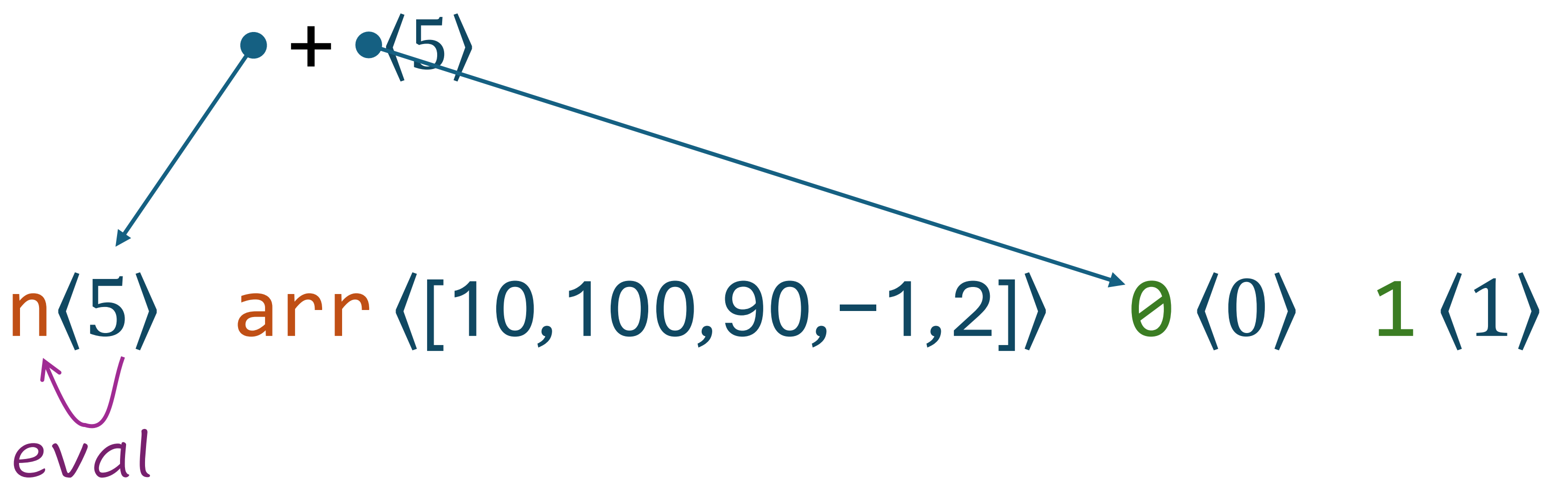

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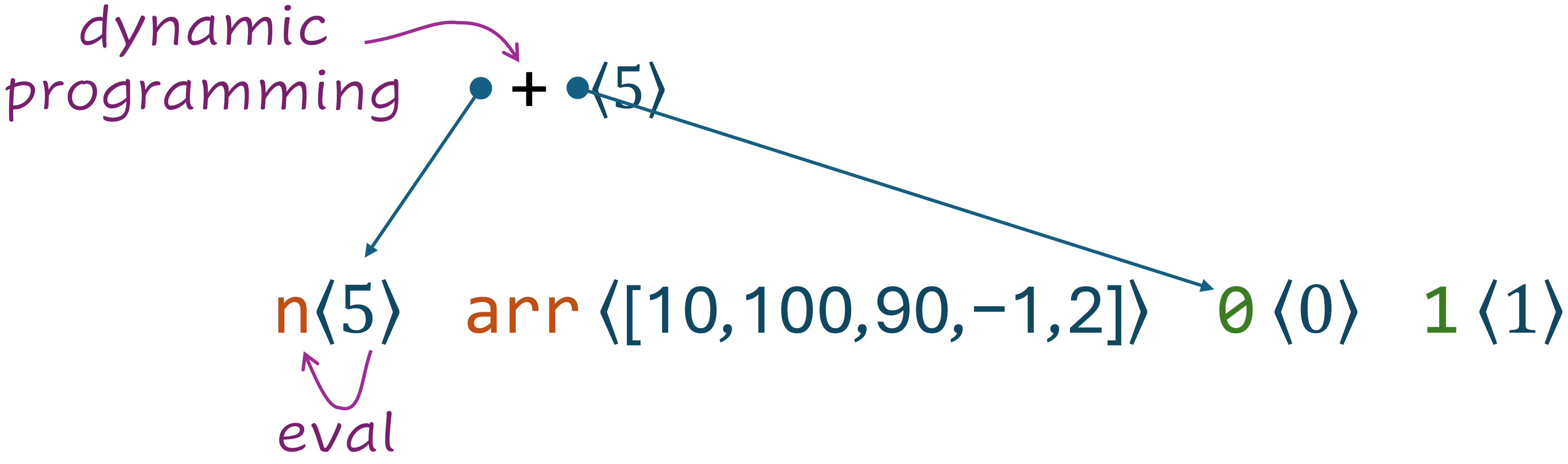

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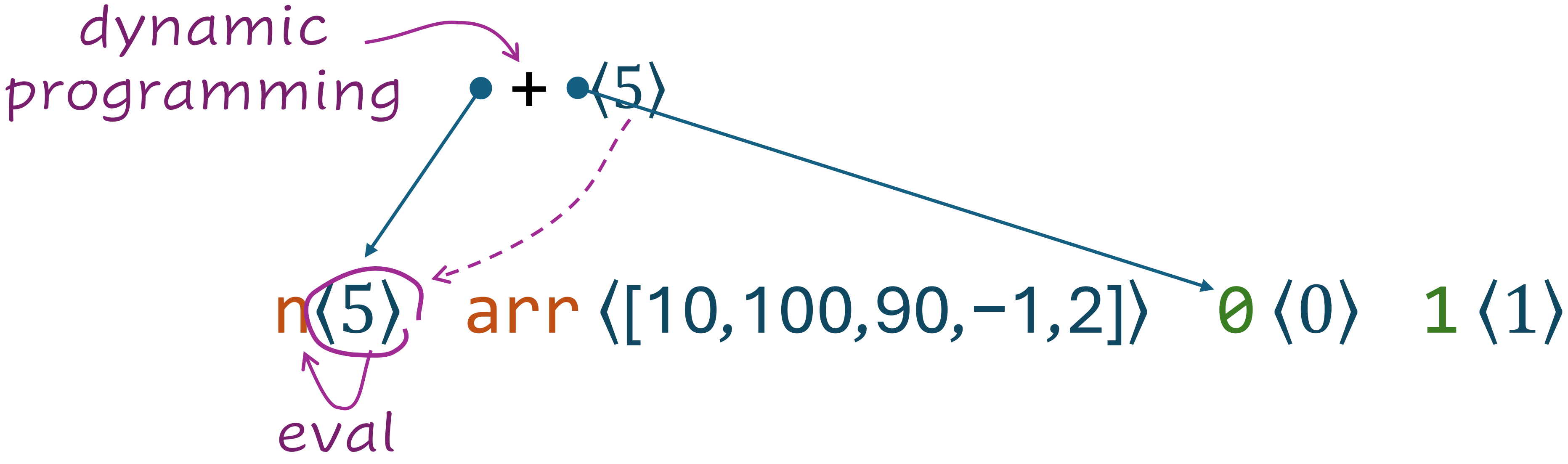

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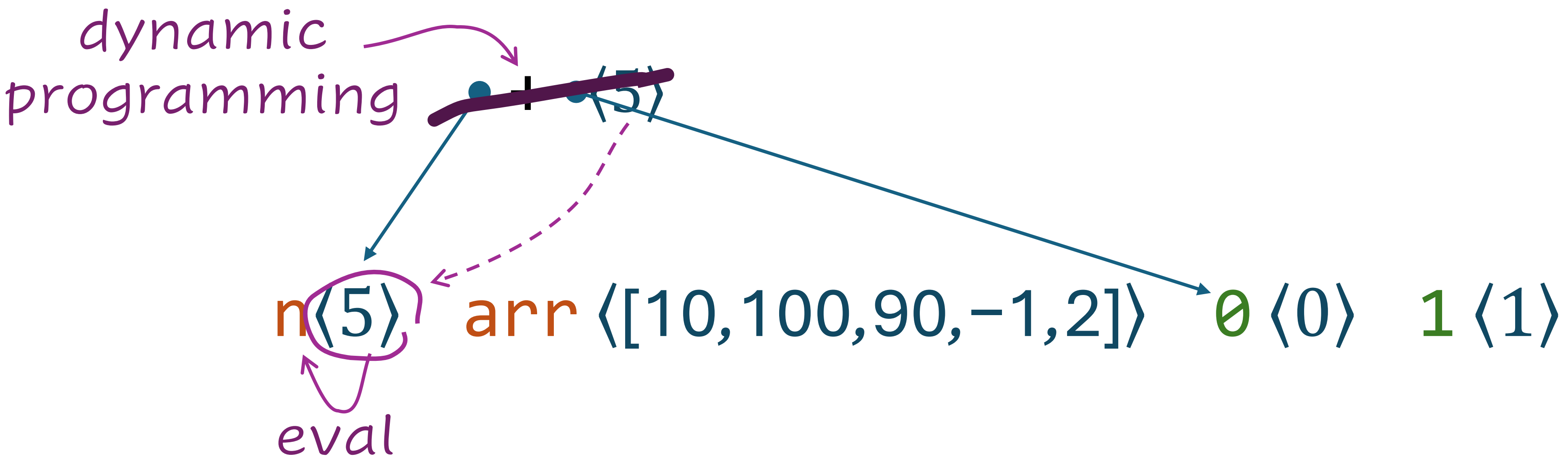

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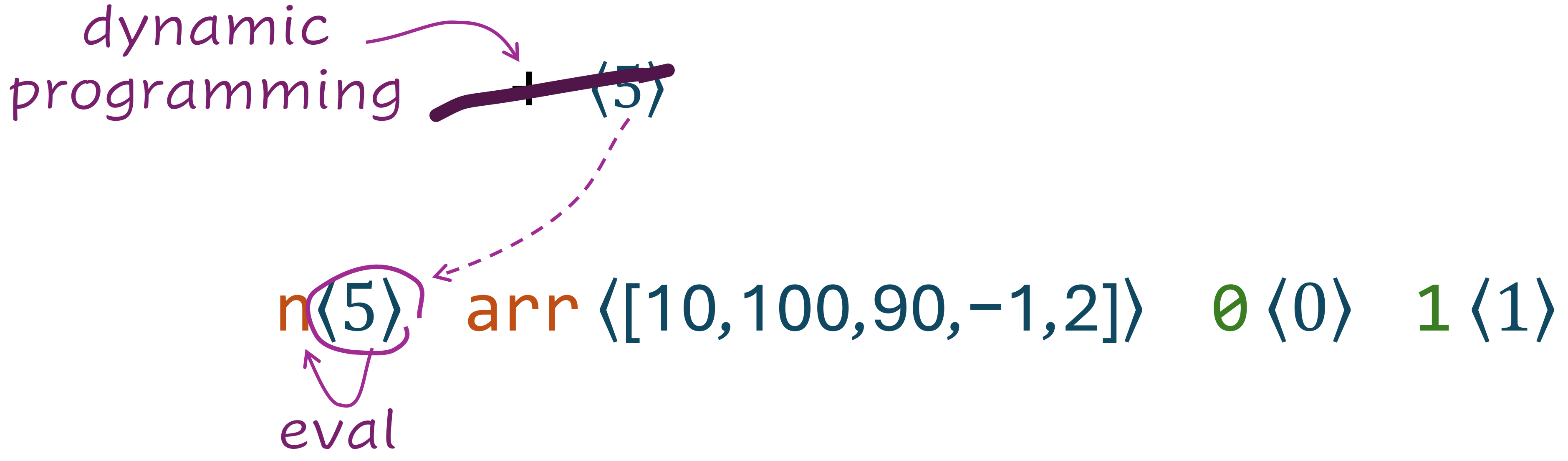

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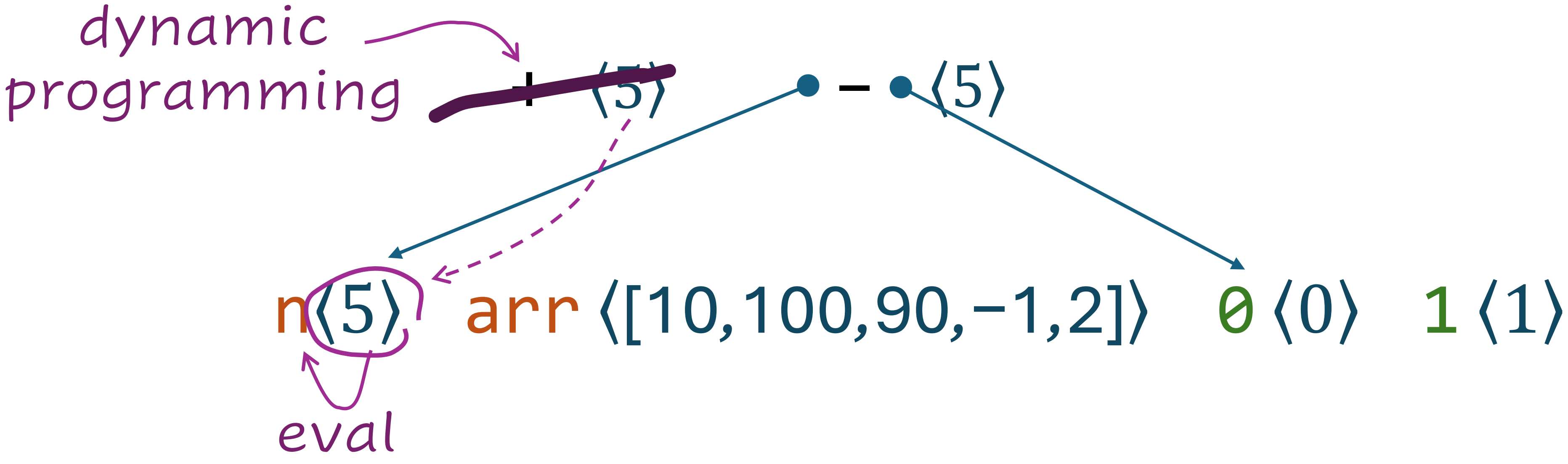

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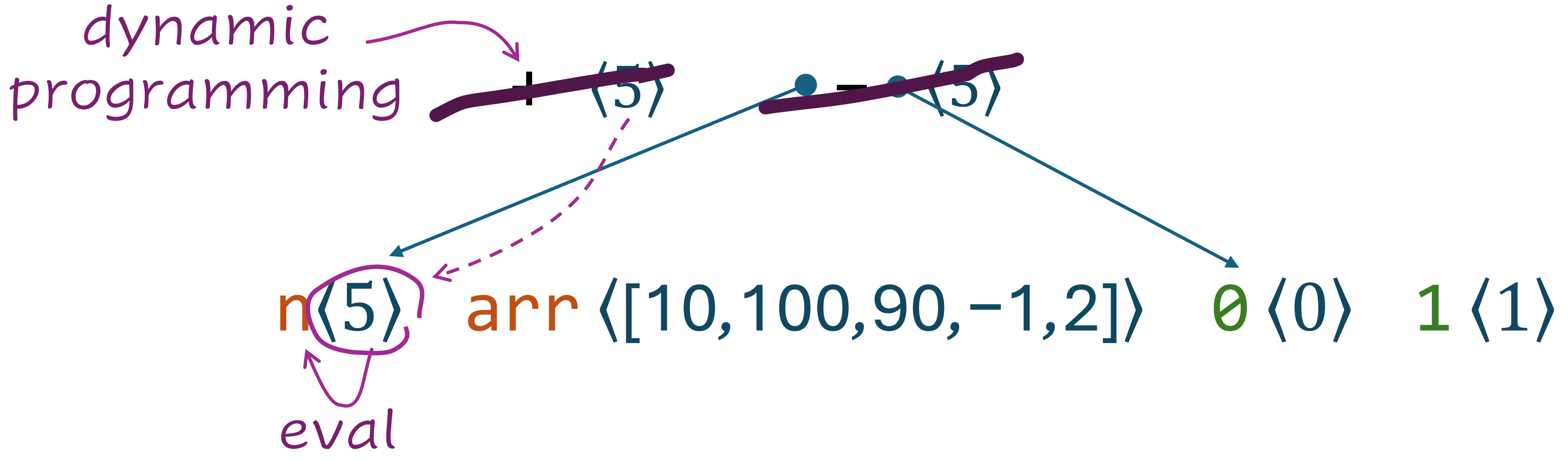

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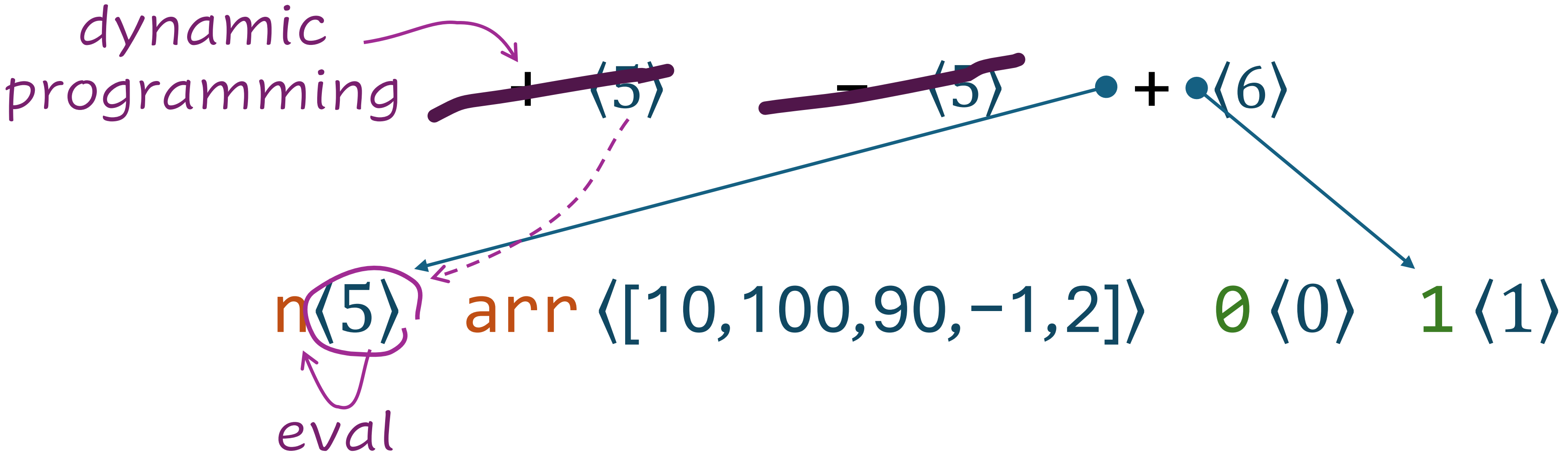

Observational Equivalence and Mutations

Can we build `arr.sort()[0] + arr[n-1]`?

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arr ⟨[10, 100, 90, −1, 2]⟩

Observational Equivalence and Mutations

Can we build `arr.sort()[0] + arr[n-1]`?

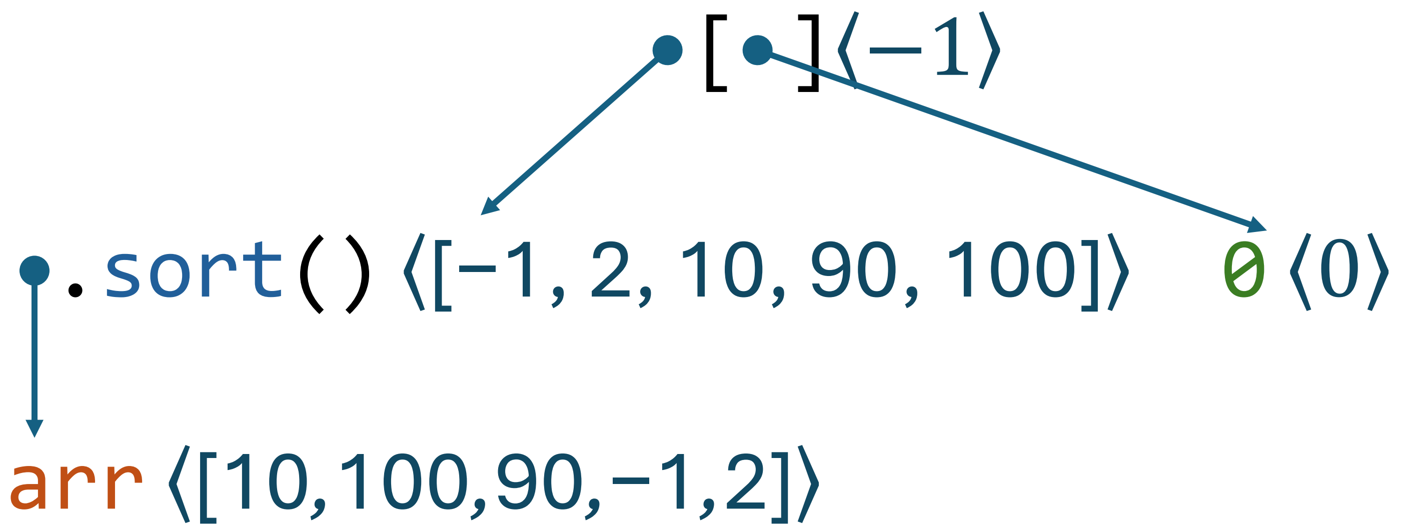
`.sort()` ⟨[−1, 2, 10, 90, 100]⟩

↓

`arr` ⟨[10,100,90,−1,2]⟩

Observational Equivalence and Mutations

Can we build `arr.sort()[0] + arr[n-1]`?



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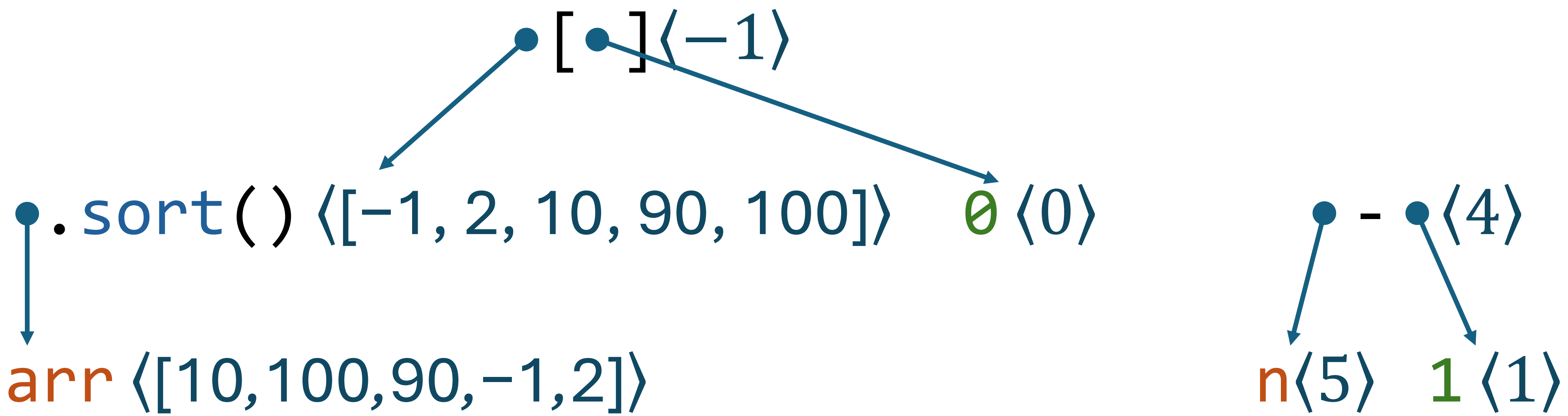

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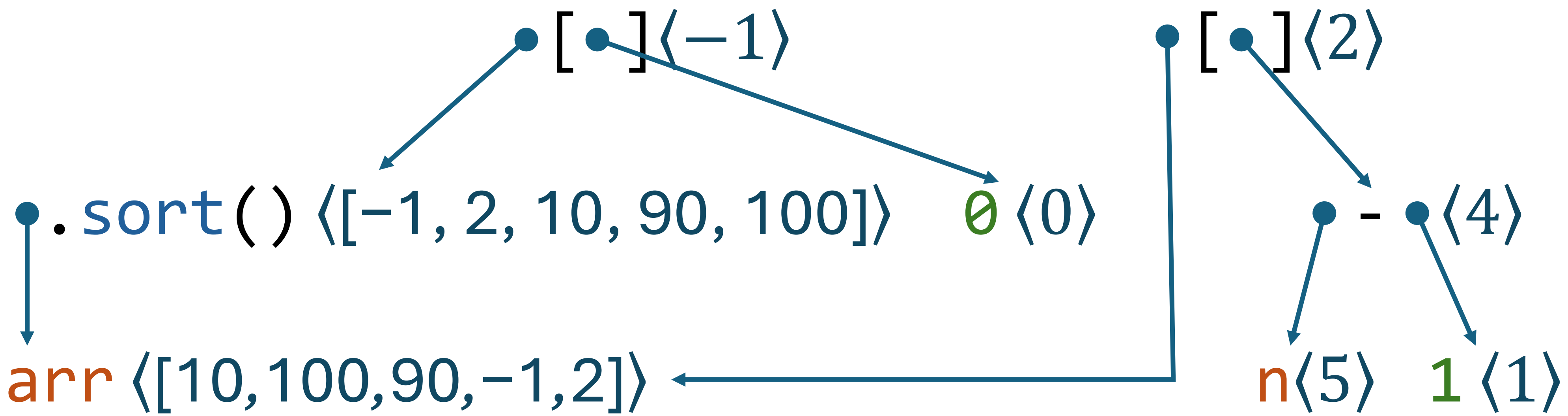

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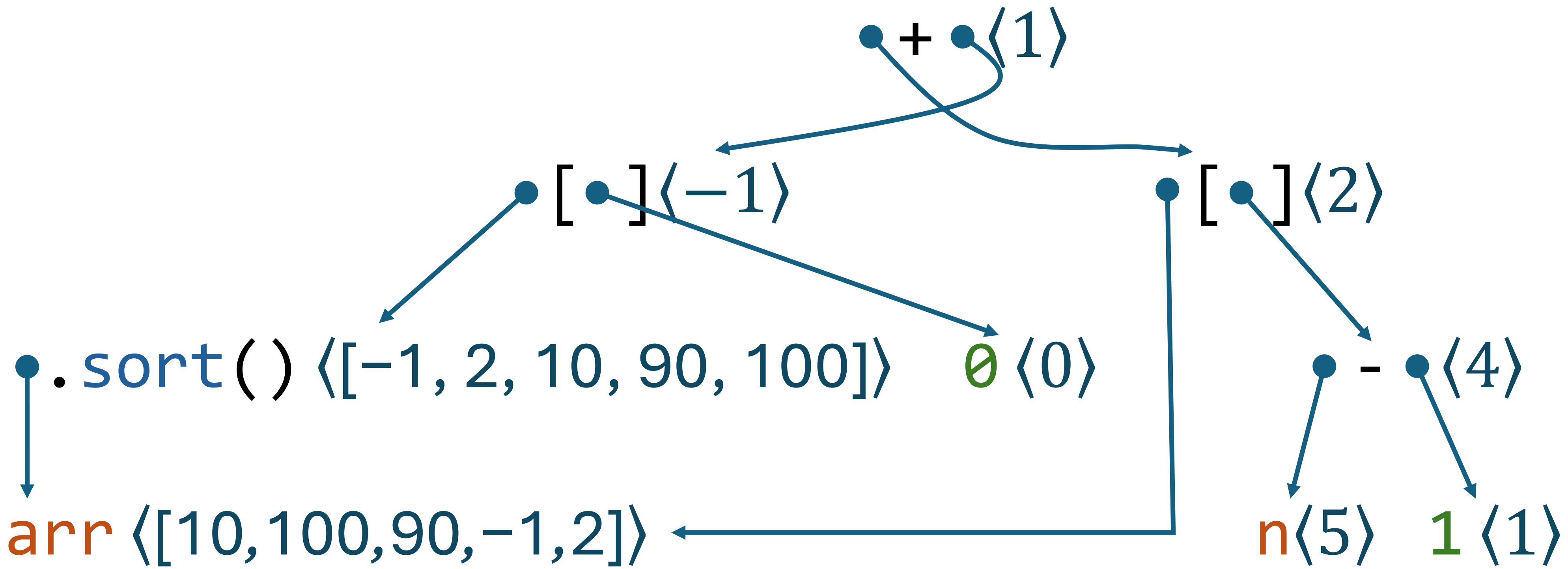

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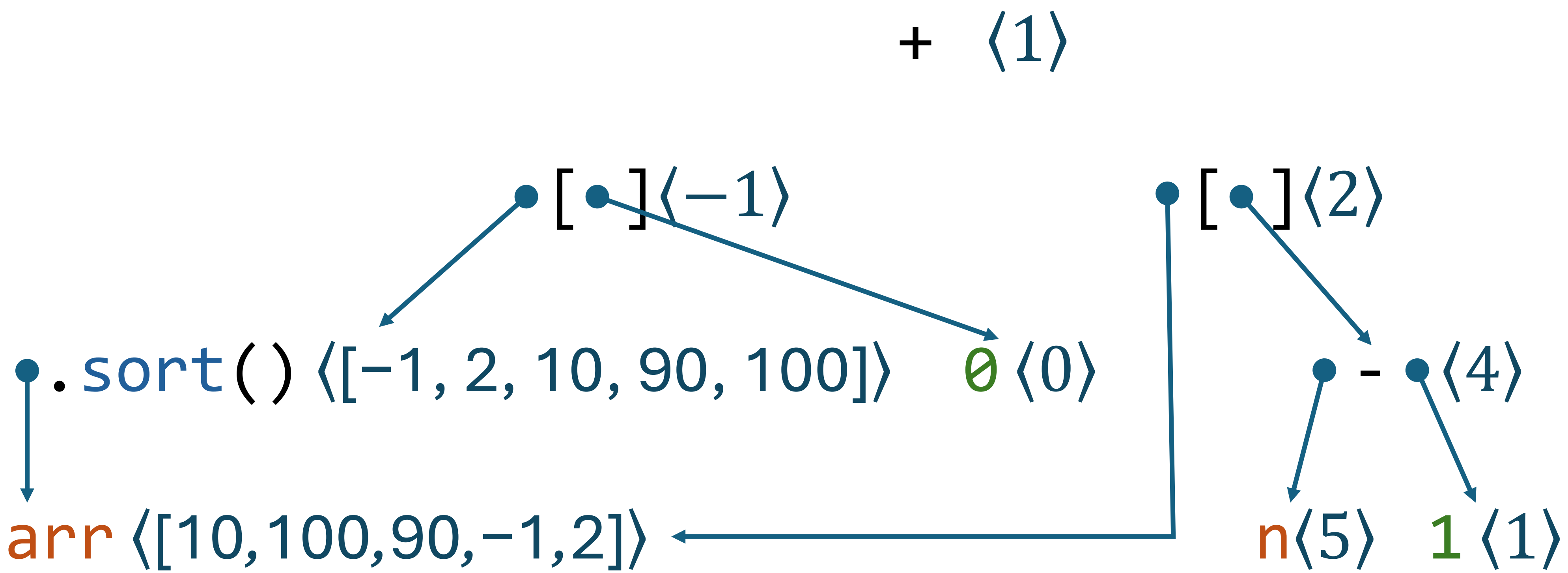

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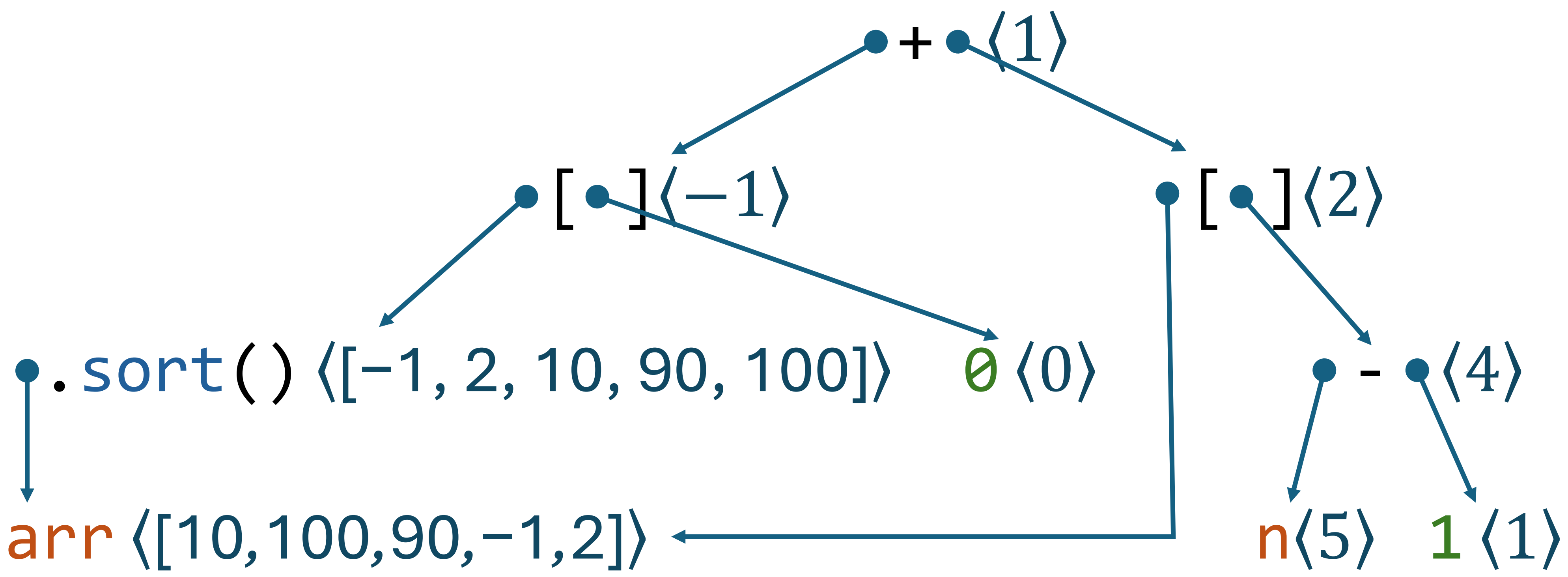

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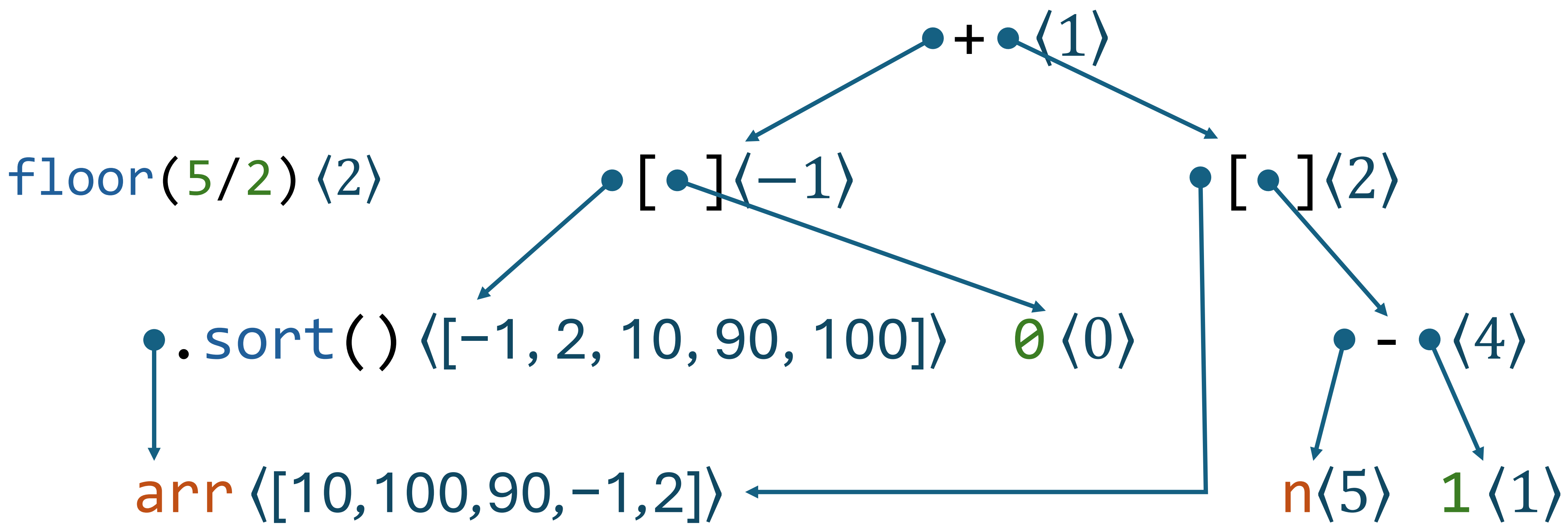

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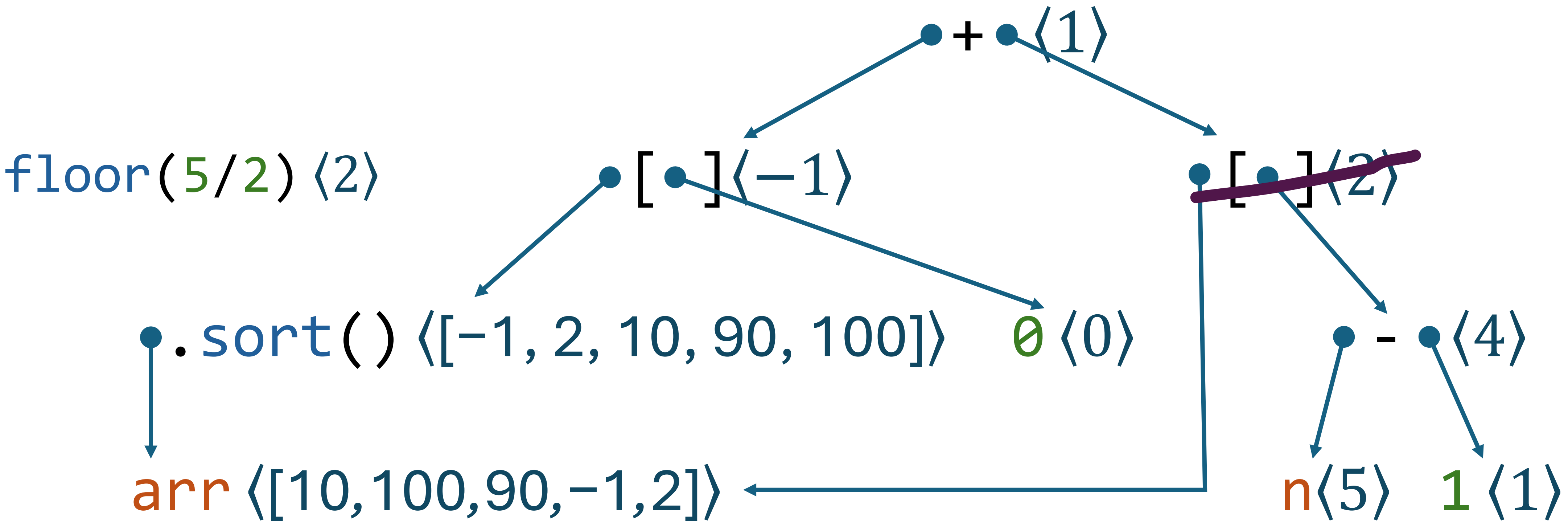

Let’s fix it!

Solution: ***S****ide-effects in* ***OB****servational* ***EQ****uivalence*

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1) Add before- and after-states to representation

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1) Add before- and after-states to representation
2) Trim states to bare necessities

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Solution: ***S****ide-effects in* ***OB****servational* ***EQ****uivalence*

1) Add before- and after-states to representation
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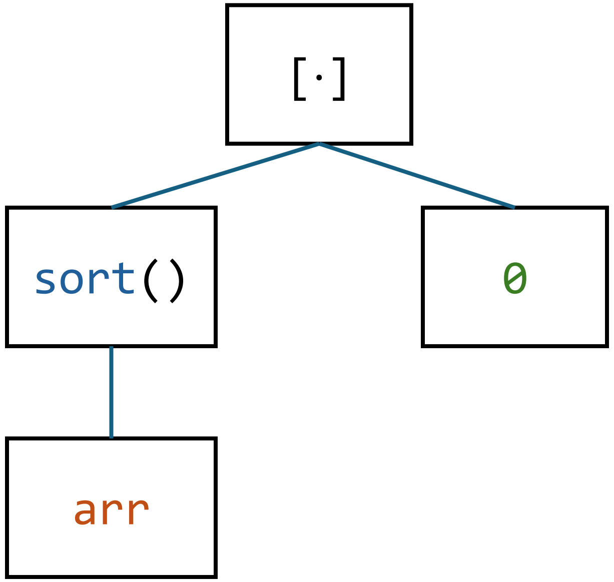
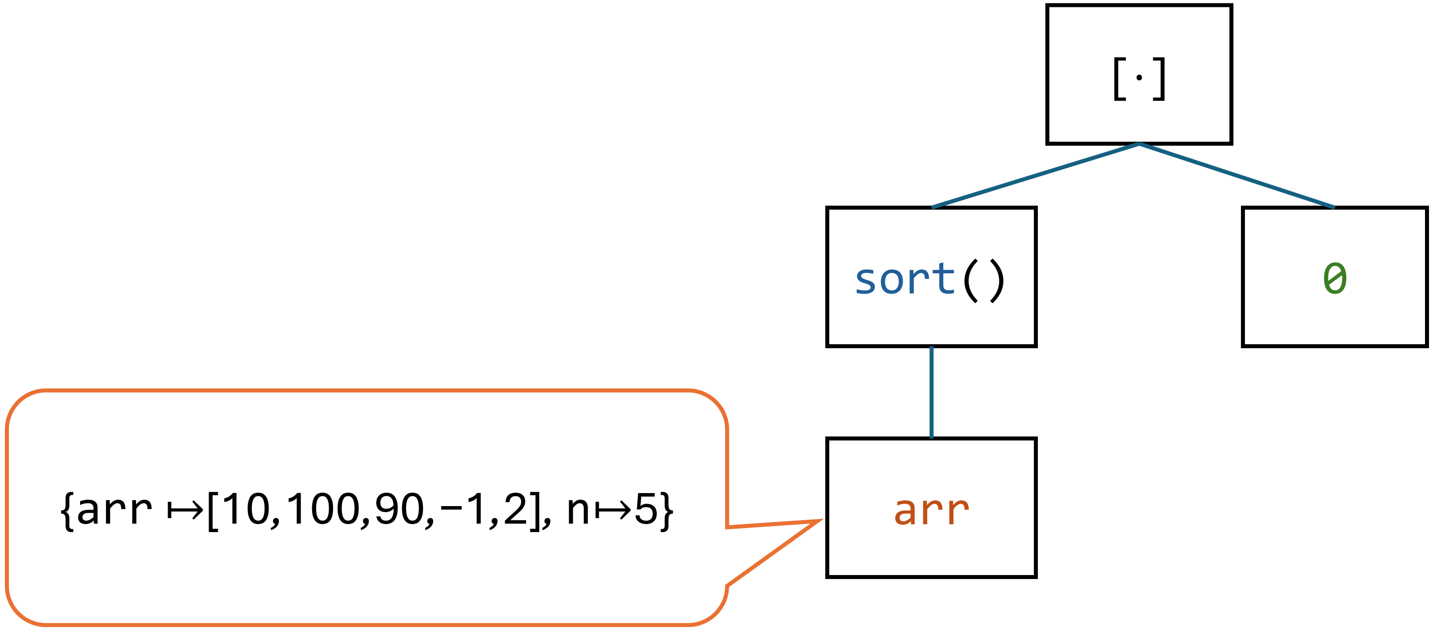
Correct enumeration with mutations!

Function arguments and sequencing



Function arguments and sequencing



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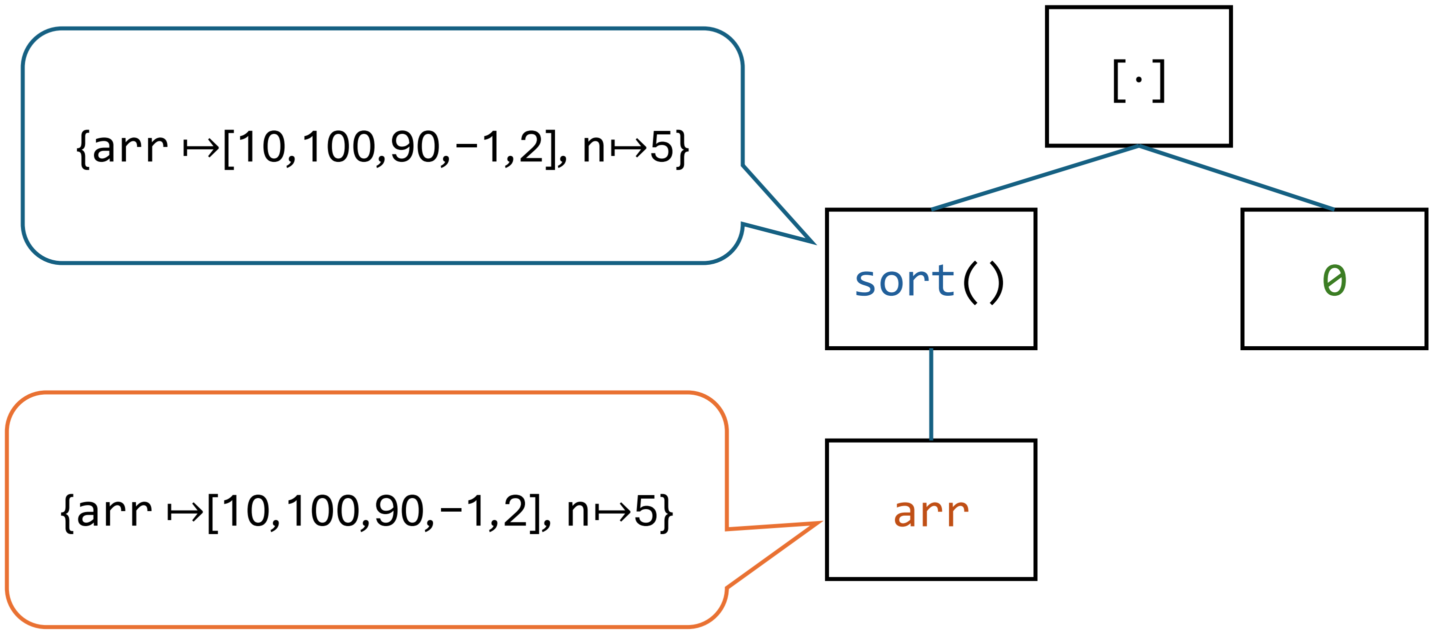

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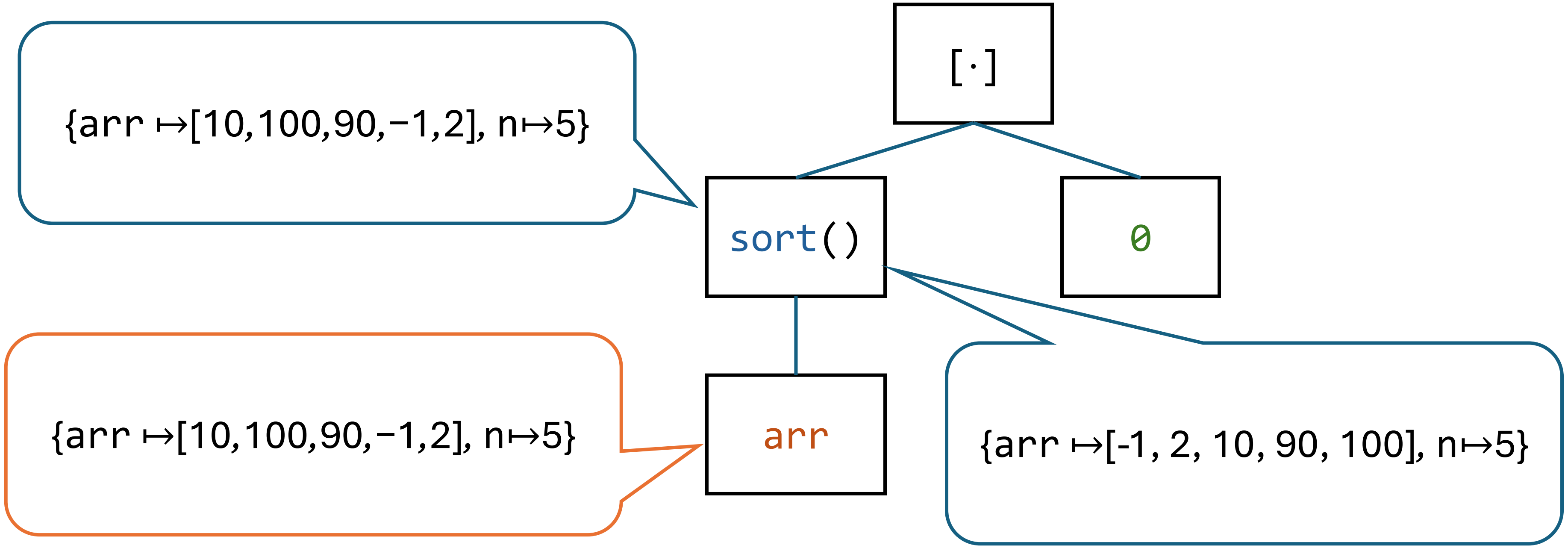

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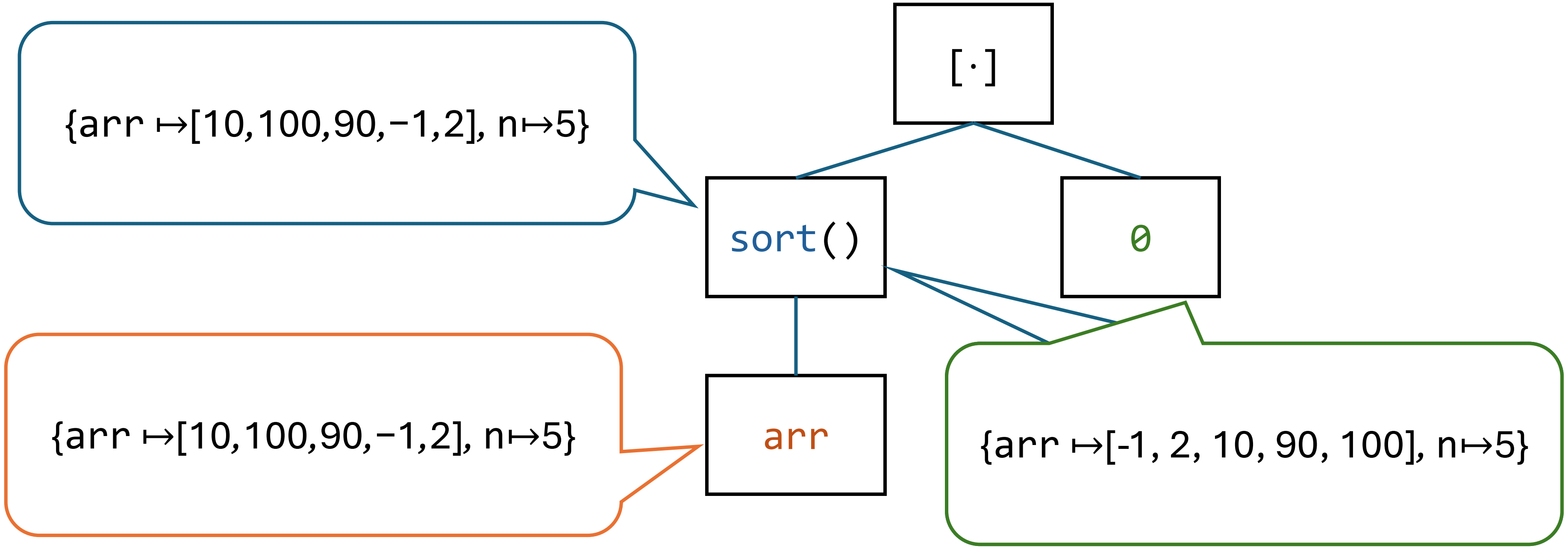

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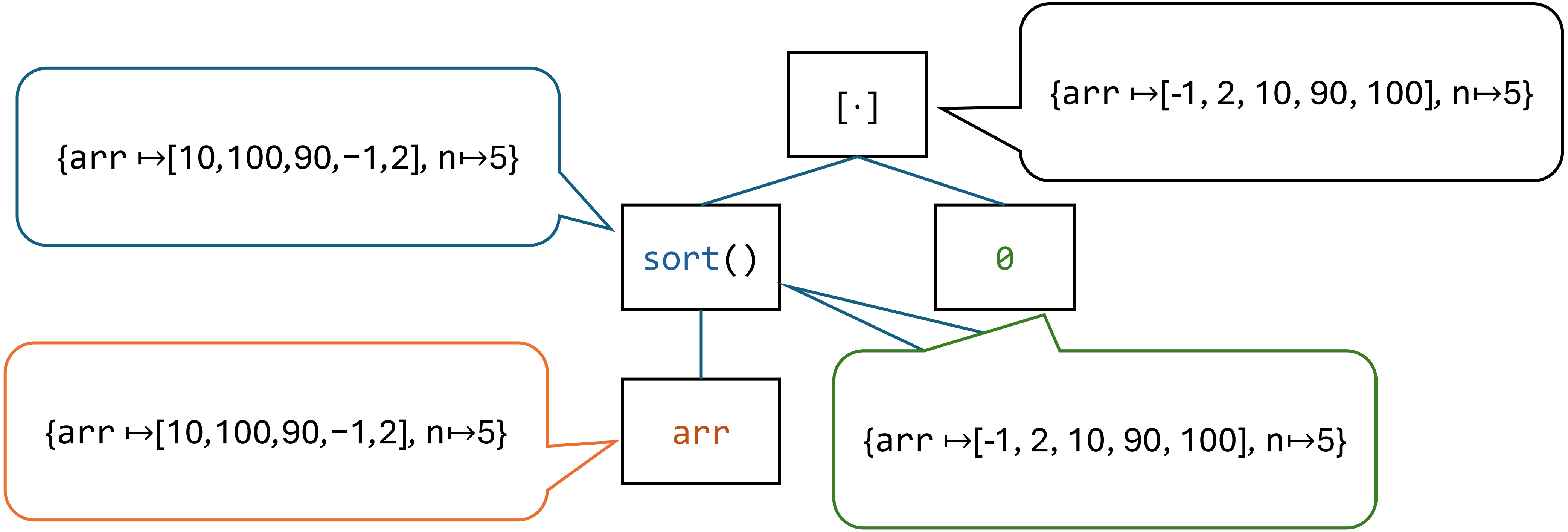

Key idea 1: triples

$$\underbrace{\{x \mapsto v_x, y \mapsto v_y, \ldots\}}_{\text{precondition}}\ p\ \underbrace{\{x \mapsto v'_x, y \mapsto v'_y, \ldots; r\}}_{\text{postcondition}}$$

Key idea 1: triples

state after eval

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\; p \;\{x \mapsto v'_x, y \mapsto v'_y, \ldots; r\}$$

precondition

postcondition

Key idea 1: triples

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\ p\ \{x \mapsto v'_x, y \mapsto v'_y, \ldots; r\}$$

Key idea 1: triples

state after eval

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\ p\ \{x \mapsto v'_x, y \mapsto v'_y, \ldots; r\}$$

precondition

postcondition

result

{arr ↦[10,100,90,−1,2]} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100]; -1}

Key idea 1: triples

state after eval

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\ p\ \{x \mapsto v'_x, y \mapsto v'_y, \ldots; r\}$$

precondition

postcondition

result

{arr ↦[10,100,90,−1,2]} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100]; -1}

side effect!

Key idea 1: triples

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\; p\; \{x \mapsto v'_x, y \mapsto v'_y, \ldots; r\}$$

valid sequence

$$\{x \mapsto {v'}_x, y \mapsto {v'}_y, \ldots\}\; p'\; \{x \mapsto v''_x, y \mapsto v''_y, \ldots; r'\}$$

Key idea 1: triples

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\ p;$$

$$p'\ \{x \mapsto v_x'', y \mapsto v_y'', \ldots; r'\}$$

Key idea 1: triples

initial state — *arr is sorted*

{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

{arr ↦[-1, 2, 10, 90, 100], n↦5}`arr[n-1]` {arr ↦[-1, 2, 10, 90, 100], n↦5;100}

arr is sorted

Key idea 1: triples

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{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

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{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

arr is sorted

valid sequence

{arr ↦[-1, 2, 10, 90, 100], n↦5}`arr[n-1]` {arr ↦[-1, 2, 10, 90, 100], n↦5;100}

arr is sorted

✓ Apply `f(?,?)`

Key idea 1: triples

initial state

arr is sorted

{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

<u>**in**</u>valid sequence!

{arr ↦[10,100,90,−1,2], n↦5} `arr[n-1]` {arr ↦[10,100,90,−1,2], n↦5;2}

initial state

initial state

Key idea 1: triples

initial state — *arr is sorted*

{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

invalid sequence!

{arr ↦[10,100,90,−1,2], n↦5} `arr[n-1]` {arr ↦[10,100,90,−1,2], n↦5;2}

initial state — *initial state*

✘ Apply `f(?,?)`

Key idea 1: triples

{arr ↦[10,100,90,−1,2], n↦5} `arr[n-1]` {arr ↦[10,100,90,−1,2], n↦5;2}

valid sequence

{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

Key idea 1: triples

{arr ↦[10,100,90,−1,2], n↦5} `arr[n-1]` {arr ↦[10,100,90,−1,2], n↦5;2}

valid sequence

{arr ↦[10,100,90,−1,2], n↦5} `arr.sort()[0]` {arr ↦[-1, 2, 10, 90, 100], n↦5;-1}

✓ Apply `f(?,?)`

Key idea 1: triples

$$p\langle r\rangle$$

Key idea 1: triples

$$\langle r \rangle$$

Key idea 1: triples

$$\langle r \rangle$$

$$\{initial\ state\}\ p\ \{initial\ state; r\}$$

Key idea 1: triples

$$\cancel{p}\langle r \rangle$$

$$\{initial\ state\}\ p\ \{initial\ state; r\}$$

$$\{x \mapsto v_x, y \mapsto v_y, \dots\}\ p\ \{x \mapsto v_x', y \mapsto v_y', \dots; r\}$$

Key idea 1: triples

$$\langle r\rangle$$

$$\{initial\ state\}\ p\ \{initial\ state; r\}$$

$$\{x \mapsto v_x, y \mapsto v_y, \ldots\}\ \{x \mapsto v_x', y \mapsto v_y', \ldots; r\}$$

Is this stupid?

initial state

{arr ↦[10,100,90,−1,2], n↦5} 0 {arr ↦[10,100,90,−1,2], n↦5; 0}

Is this stupid?

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{arr ↦[10,100,90,−1,2], n↦5} 0 {arr ↦[10,100,90,−1,2], n↦5; 0}

arr is sorted

{arr ↦[-1, 2, 10, 90, 100], n↦5} 0 {arr ↦[-1, 2, 10, 90, 100], n↦5; 0}

Is this stupid?

initial state

{arr ↦[10,100,90,−1,2], n↦5} `0` {arr ↦[10,100,90,−1,2], n↦5; 0}

arr is sorted

{arr ↦[-1, 2, 10, 90, 100], n↦5} `0` {arr ↦[-1, 2, 10, 90, 100], n↦5; 0}

arr is empty

{arr ↦[], n↦5} `0` {arr ↦[], n↦5; 0}

Is this stupid?

initial state

{arr ↦ [10,100,90,−1,2], n ↦ 5} `0` {arr ↦ [10,100,90,−1,2], n ↦ 5; 0}

arr is sorted

{arr ↦ [-1, 2, 10, 90, 100], n ↦ 5} `0` {arr ↦ [-1, 2, 10, 90, 100], n ↦ 5; 0}

arr is empty

{arr ↦ [], n ↦ 5} `0` {arr ↦ [], n ↦ 5; 0}

{arr ↦ [10,100,90,−1], n ↦ 5} `0` {arr ↦ [10,100,90,−1], n ↦ 5; 0}

Separation Logic!

one part of the heap * another part of the heap

Separation Logic!

one part of the heap $*$ another part of the heap

$$\frac{\{P\}\ p\ \{Q\}}{\{P * R\}\ p\ \{Q * R\}}\ \textsc{Frame}$$

Key idea 2: local triplets using SL

$$\underbrace{\{x \mapsto v_x\ ,\ \ y \mapsto v_y, \ldots\}}_{\text{precondition}}\ p\ \{\underbrace{x \mapsto v'_x\ ,\ y \mapsto v'_y, \ldots ;\ \underbrace{r}_{\text{result}}}_{\text{postcondition}}\}$$

Key idea 2: local triplets using SL

$$\underbrace{\{x \mapsto v_x * \; y \mapsto v_y, \ldots\}}_{\text{precondition}} \; p \; \underbrace{\{x \mapsto v'_x * y \mapsto v'_y, \ldots; r\}}_{\text{postcondition}}$$

result

Key idea 2: local triplets using SL

$$\{x \mapsto v_x * y \mapsto v_y, \ldots\}\; p \;\{x \mapsto v'_x * y \mapsto v'_y, \ldots ; r\}$$

precondition

postcondition

result

$\{arr \mapsto [-1, 2, 10, 90, 100] * n \mapsto 5\}$ `n - 1` $\{arr \mapsto [-1, 2, 10, 90, 100] * n \mapsto 5; 4\}$

Key idea 2: local triplets using SL

$$\{x \mapsto v_x * \ y \mapsto v_y, \ldots\}\ p\ \{x \mapsto v_x' * y \mapsto v_y', \ldots; r\}$$

precondition

postcondition

result

{~~arr ↦ [-1, 2, 10, 90, 100]~~ * n↦5} n - 1 {~~arr ↦ [-1, 2, 10, 90, 100]~~ * n↦5; 4}

Key idea 2: local triplets using SL

$$\{\underbrace{x \mapsto v_x * y \mapsto v_y, \dots}_{\text{precondition}}\}\ p\ \{\underbrace{x \mapsto v'_x * y \mapsto v'_y, \dots;}_{\text{postcondition}}\ \underset{\uparrow\ \text{result}}{r}\}$$

{~~arr ↦[-1, 2, 10, 90, 100]~~ * n↦5} `n - 1` {~~arr ↦[-1, 2, 10, 90, 100]~~ * n↦5; 4}

{arr ↦[-1, 2, 10, 90, 100] * n↦5} `0` {arr ↦[-1, 2, 10, 90, 100] * n↦5; 0}

Key idea 2: local triplets using SL

$$\{x \mapsto v_x * y \mapsto v_y, \ldots\}\ p\ \{x \mapsto v_x' * y \mapsto v_y', \ldots; r\}$$

precondition

postcondition

result

{~~arr ↦ [-1, 2, 10, 90, 100]~~ * n↦5} n – 1 {~~arr ↦ [-1, 2, 10, 90, 100]~~ * n↦5; 4}

{~~arr ↦ [-1, 2, 10, 90, 100] * n↦~~5} 0 {~~arr ↦ [-1, 2, 10, 90, 100] * n↦~~5; 0}

Key idea 2: local triplets using SL

$$\{x \mapsto v_x * \ y \mapsto v_y, \dots\}\ p\ \{x \mapsto v'_x * y \mapsto v'_y, \dots ; r\}$$

precondition

postcondition

result

{arr ↦ [-1, 2, 10, 90, 100] * n↦5} n - 1 {arr ↦ [-1, 2, 10, 90, 100] * n↦5; 4}

{arr ↦ [-1, 2, 10, 90, 100] {emp} 0 {emp; 0} 1, 2, 10, 90, 100] * n↦5; 0}

Separation Logic!

one part of the heap $*$ another part of the heap

$$\frac{\{P\}\ p\ \{Q\}}{\{P * R\}\ p\ \{Q * R\}}\ \textsc{Frame}$$

Key idea 2: local triplets using SL

$$\frac{\{P\}\ p\ \{Q; r\}}{\{P * R\}\ p\ \{Q * R; r\}}\ \textsc{Frame}$$

Key idea 2: local triplets using SL

$$\frac{\{P\}\ p\ \{Q; r\}}{\{P * R\}\ p\ \{Q * R; r\}}\ \textsc{Frame}$$

$$\frac{\{P_1\}\ p_1\ \{P_2; r_1\}\ \{P_2\}\ p_2\ \{P_3; r_2\} \cdots \{P_k\}\ p_k\ \{P_{k+1}; r_k\} \quad (c(r_1, \ldots, r_k), P_{k+1}) \rightarrow (r, Q)}{\{P_1\}\ c(p_1, \ldots, p_k)\ \{Q; r\}}\ \textsc{Eval}$$

Key idea 2: local triplets using SL

$$\frac{\{P\}\ p\ \{Q; r\}}{\{P * R\}\ p\ \{Q * R; r\}}\ \textsc{Frame}$$

$$\frac{\{P_1\}\ p_1\ \{P_2; r_1\}\ \{P_2\}\ p_2\ \{P_3; r_2\} \cdots \{P_k\}\ p_k\ \{P_{k+1}; r_k\} \quad (c(r_1, \ldots, r_k), P_{k+1}) \to (r, Q)}{\{P_1\}\ c(p_1, \ldots, p_k)\ \{Q; r\}}\ \textsc{Eval}$$

interpreter

Program proofs with EVAL and FRAME

[-1, 2, 10, 90, 100]

{arr ↦ v_{arr}} arr {arr ↦ v_{arr}; v_{arr}}

{emp} 1 {emp; 1}

{n↦ 5} n {n↦ 5; 5}

Program proofs with Eval and Frame

[-1, 2, 10, 90, 100]

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\, v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\}$

$$\frac{\{emp\}\ \texttt{1}\ \{emp;\, 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5;\, 1\}}\ \textsc{Frame}$$

Program proofs with EVAL and FRAME

[-1, 2, 10, 90, 100]

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\ v_{arr}\}$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\ 5\} \qquad \dfrac{\{emp\}\ \texttt{1}\ \{emp;\ 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5;\ 1\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5;\ 4\}}\ \textsc{Eval}$$

Program proofs with Eval and Frame

[-1, 2, 10, 90, 100]

$$\frac{\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\, v_{arr}\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\, v_{arr}\}}\ \textsc{Frame}$$

$$\frac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\} \qquad \dfrac{\{emp\}\ \texttt{1}\ \{emp;\, 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5;\, 1\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5;\, 4\}}\ \textsc{Eval}$$

Program proofs with EVAL and FRAME

[-1, 2, 10, 90, 100]

$$\frac{\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\, v_{arr}\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\, v_{arr}\}}\ \textsc{Frame}$$

$$\frac{\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\} \qquad \dfrac{\{emp\}\ \texttt{1}\ \{emp;\, 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5;\, 1\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5;\, 4\}}\ \textsc{Eval}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\, 4\}}\ \textsc{Frame}$$

Program proofs with EVAL and FRAME

[-1, 2, 10, 90, 100]

$$\dfrac{\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5; v_{arr}\}}\ \textsc{Frame}$$

$$\dfrac{\{emp\}\ \texttt{1}\ \{emp; 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5; 1\}}\ \textsc{Frame}$$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\} \qquad \{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5; 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5; 4\}}\ \textsc{Eval}$$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5; 4\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5; 4\}}\ \textsc{Frame}$$

$$\dfrac{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5; v_{arr}\} \qquad \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5; 4\}}{\quad}\ \textsc{Eval}$$

Program proofs with EVAL and FRAME

[-1, 2, 10, 90, 100]

$$\dfrac{\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\ v_{arr}\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\ v_{arr}\}}\ \textsc{Frame}$$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\ 5\} \qquad \dfrac{\{emp\}\ \texttt{1}\ \{emp;\ 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5;\ 1\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5;\ 4\}}\ \textsc{Eval}$$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{n} \mapsto 5;\ 4\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\ 4\}}\ \textsc{Frame}$$

$$\dfrac{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr}\ \{\ldots;\ v_{arr}\} \qquad \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{n - 1}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\ 4\}}{\{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5\}\ \texttt{arr[n - 1]}\ \{\texttt{arr} \mapsto v_{arr} * \texttt{n} \mapsto 5;\ 100\}}\ \textsc{Eval}$$

Bottom-up proof search

$\{\mathtt{arr} \mapsto v_{arr}\}$ `arr` $\{\mathtt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\mathtt{n} \mapsto 5\}$ `n` $\{\mathtt{n} \mapsto 5; 5\}$

$\{emp\}$ `1` $\{emp; 1\}$

$\{emp\}$ `0` $\{emp; 0\}$

Bottom-up proof search

apply **+**

$\{\texttt{arr} \mapsto v_{arr}\}$ `arr` $\{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}$ `n` $\{\texttt{n} \mapsto 5; 5\}$

$\{emp\}$ `1` $\{emp; 1\}$

$\{emp\}$ `0` $\{emp; 0\}$

Bottom-up proof search

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1 2

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

apply **+**

Bottom-up proof search

$\{\texttt{arr} \mapsto v_{arr}\}$ arr $\{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}$ n $\{\texttt{n} \mapsto 5; 5\}$ 1 2

$\{emp\}$ 1 $\{emp; 1\}$

$\{emp\}$ 0 $\{emp; 0\}$

apply **+**

$\{\texttt{n} \mapsto 5\}$ n $\{\texttt{n} \mapsto 5; 5\}$ $\{\texttt{n} \mapsto 5\}$ n $\{\texttt{n} \mapsto 5; 5\}$

Bottom-up proof search

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1 2

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

apply **+**

$$\frac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\} \quad \{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}}{\{\texttt{n} \mapsto 5\}\ \texttt{n} + \texttt{n}\ \{\texttt{n} \mapsto 5; 10\}}\ \textsc{Eval}$$

Bottom-up proof search

apply **+**

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1 2

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n} + \texttt{n}\ \{\texttt{n} \mapsto 5; 10\}$

Bottom-up proof search

apply **+**

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1

$\{emp\}\ \texttt{1}\ \{emp; 1\}$ 2

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n} + \texttt{n}\ \{\texttt{n} \mapsto 5; 10\}$

Bottom-up proof search

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1

$\{emp\}\ \texttt{1}\ \{emp; 1\}$ 2

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n + n}\ \{\texttt{n} \mapsto 5; 10\}$

apply **+**

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$

Bottom-up proof search

apply +

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1

$\{emp\}\ \texttt{1}\ \{emp; 1\}$ 2

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ +\ \texttt{n}\ \{\texttt{n} \mapsto 5; 10\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\} \quad \dfrac{\{emp\}\ \texttt{1}\ \{emp; 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5; 1\}}\ \textsc{Frame}$

Bottom-up proof search

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ 1

$\{emp\}\ \texttt{1}\ \{emp; 1\}$ 2

$\{emp\}\ \texttt{0}\ \{emp; 0\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n + n}\ \{\texttt{n} \mapsto 5; 10\}$

apply **+**

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\} \quad \dfrac{\{emp\}\ \texttt{1}\ \{emp; 1\}}{\{\texttt{n} \mapsto 5\}\ \texttt{1}\ \{\texttt{n} \mapsto 5; 1\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n + 1}\ \{\texttt{n} \mapsto 5; 6\}}\ \textsc{Eval}$$

Bottom-up proof search

apply **+**

$\{\mathtt{arr} \mapsto v_{arr}\}\ \mathtt{arr}\ \{\mathtt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\mathtt{n} \mapsto 5\}\ \mathtt{n}\ \{\mathtt{n} \mapsto 5; 5\}$ (1)

$\{emp\}\ \mathtt{1}\ \{emp; 1\}$ (2)

$\{emp\}\ \mathtt{0}\ \{emp; 0\}$

$\{\mathtt{n} \mapsto 5\}\ \mathtt{n}\ +\ \mathtt{n}\ \{\mathtt{n} \mapsto 5; 10\}$

$\{\mathtt{n} \mapsto 5\}\ \mathtt{n}\ +\ \mathtt{1}\ \{\mathtt{n} \mapsto 5; 6\}$

Bottom-up proof search

apply **+**

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ (1)

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

$\{emp\}\ \texttt{0}\ \{emp; 0\}$ (2)

$\{\texttt{n} \mapsto 5\}\ \texttt{n + n}\ \{\texttt{n} \mapsto 5; 10\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n + 1}\ \{\texttt{n} \mapsto 5; 6\}$

Bottom-up proof search

apply **+**

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\, v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\}$ 1

$\{emp\}\ \texttt{1}\ \{emp;\, 1\}$

$\{emp\}\ \texttt{0}\ \{emp;\, 0\}$ 2

$\{\texttt{n} \mapsto 5\}\ \texttt{n + n}\ \{\texttt{n} \mapsto 5;\, 10\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n + 1}\ \{\texttt{n} \mapsto 5;\, 6\}$

$\{emp\}\ \texttt{0}\ \{emp;\, 0\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\}$

Bottom-up proof search

apply +

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr};\, v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\}$ (1)

$\{emp\}\ \texttt{1}\ \{emp;\, 1\}$

$\{emp\}\ \texttt{0}\ \{emp;\, 0\}$ (2)

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ +\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 10\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ +\ \texttt{1}\ \{\texttt{n} \mapsto 5;\, 6\}$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5;\, 5\} \qquad \dfrac{\{emp\}\ \texttt{0}\ \{emp;\, 0\}}{\{\texttt{n} \mapsto 5\}\ \texttt{0}\ \{\texttt{n} \mapsto 5;\, 0\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ +\ \texttt{0}\ \{\texttt{n} \mapsto 5;\, 5\}}\ \textsc{Eval}$$

Bottom-up proof search

$\{\mathtt{arr} \mapsto v_{arr}\}\ \mathtt{arr}\ \{\mathtt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\mathtt{n} \mapsto 5\}\ \mathtt{n}\ \{\mathtt{n} \mapsto 5; 5\}$ 1

$\{emp\}\ \mathtt{1}\ \{emp; 1\}$

$\{emp\}\ \mathtt{0}\ \{emp; 0\}$ 2

$\{\mathtt{n} \mapsto 5\}\ \mathtt{n} + \mathtt{n}\ \{\mathtt{n} \mapsto 5; 10\}$

$\{\mathtt{n} \mapsto 5\}\ \mathtt{n} + \mathtt{1}\ \{\mathtt{n} \mapsto 5; 6\}$

apply **+**

$$\dfrac{\{\mathtt{n} \mapsto 5\}\ \mathtt{n}\ \{\mathtt{n} \mapsto 5; 5\} \qquad \dfrac{\{emp\}\ \mathtt{0}\ \{emp; 0\}}{\{\mathtt{n} \mapsto 5\}\ \mathtt{0}\ \{\mathtt{n} \mapsto 5; 0\}}\ \textsc{Frame}}{\{\mathtt{n} \mapsto 5\}\ \mathtt{n} + \mathtt{0}\ \{\mathtt{n} \mapsto 5; 5\}}\ \textsc{Eval}$$

Bottom-up proof search

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ (1)

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

$\{emp\}\ \texttt{0}\ \{emp; 0\}$ (2)

$\{\texttt{n} \mapsto 5\}\ \texttt{n} + \texttt{n}\ \{\texttt{n} \mapsto 5; 10\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n} + \texttt{1}\ \{\texttt{n} \mapsto 5; 6\}$

apply **+**

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\} \qquad \dfrac{\{emp\}\ \texttt{0}\ \{emp; 0\}}{\{\texttt{n} \mapsto 5\}\ \texttt{0}\ \{\texttt{n} \mapsto 5; 0\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n} + \texttt{0}\ \{\texttt{n} \mapsto 5; 5\}}\ \textsc{Eval}$$

Bottom-up proof search

apply **+**

$\{\texttt{arr} \mapsto v_{arr}\}\ \texttt{arr}\ \{\texttt{arr} \mapsto v_{arr}; v_{arr}\}$

$\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\}$ **1**

$\{emp\}\ \texttt{1}\ \{emp; 1\}$

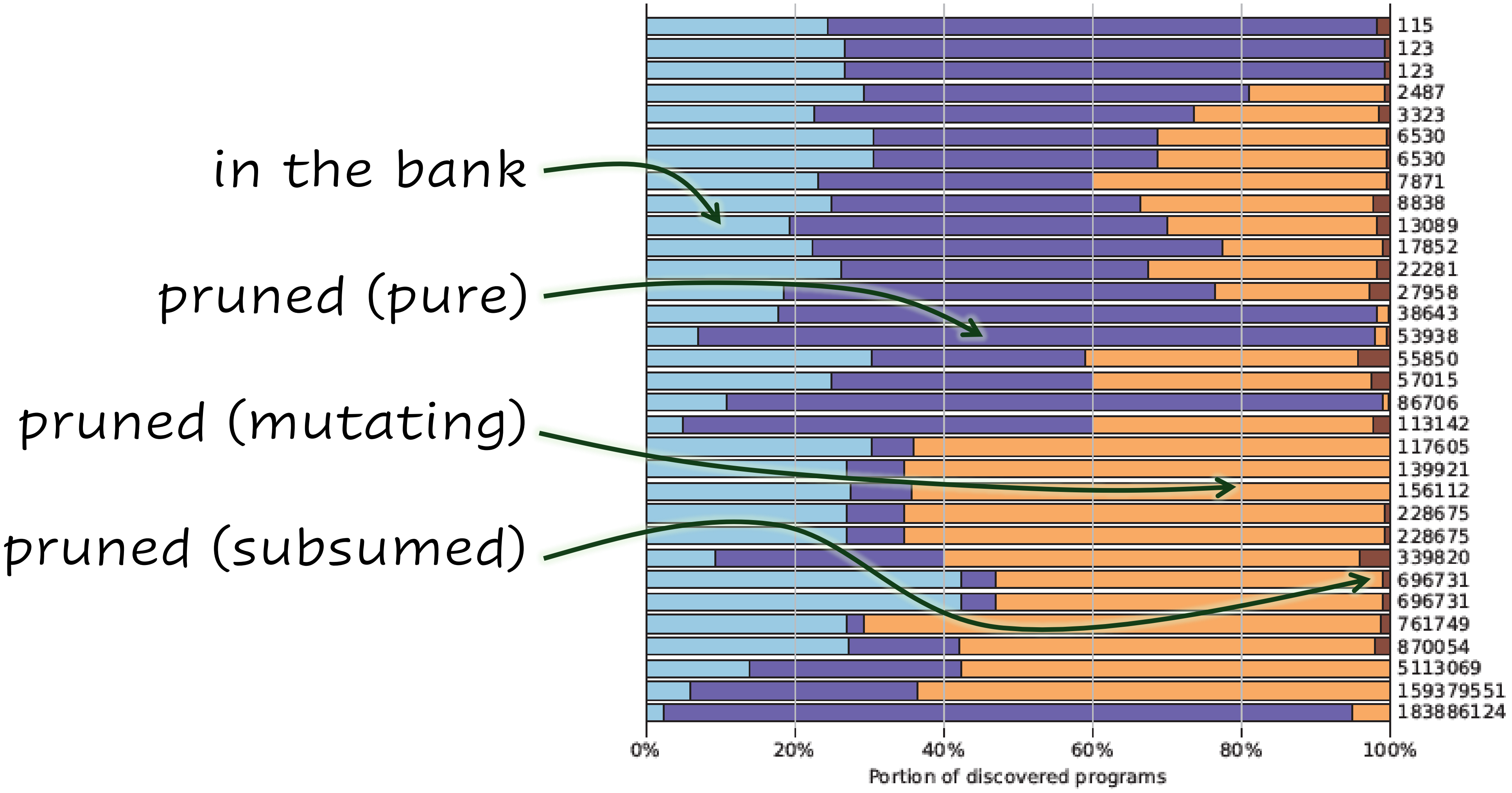
$\{emp\}\ \texttt{0}\ \{emp; 0\}$ **2**

$\{\texttt{n} \mapsto 5\}\ \texttt{n + n}\ \{\texttt{n} \mapsto 5; 10\}$

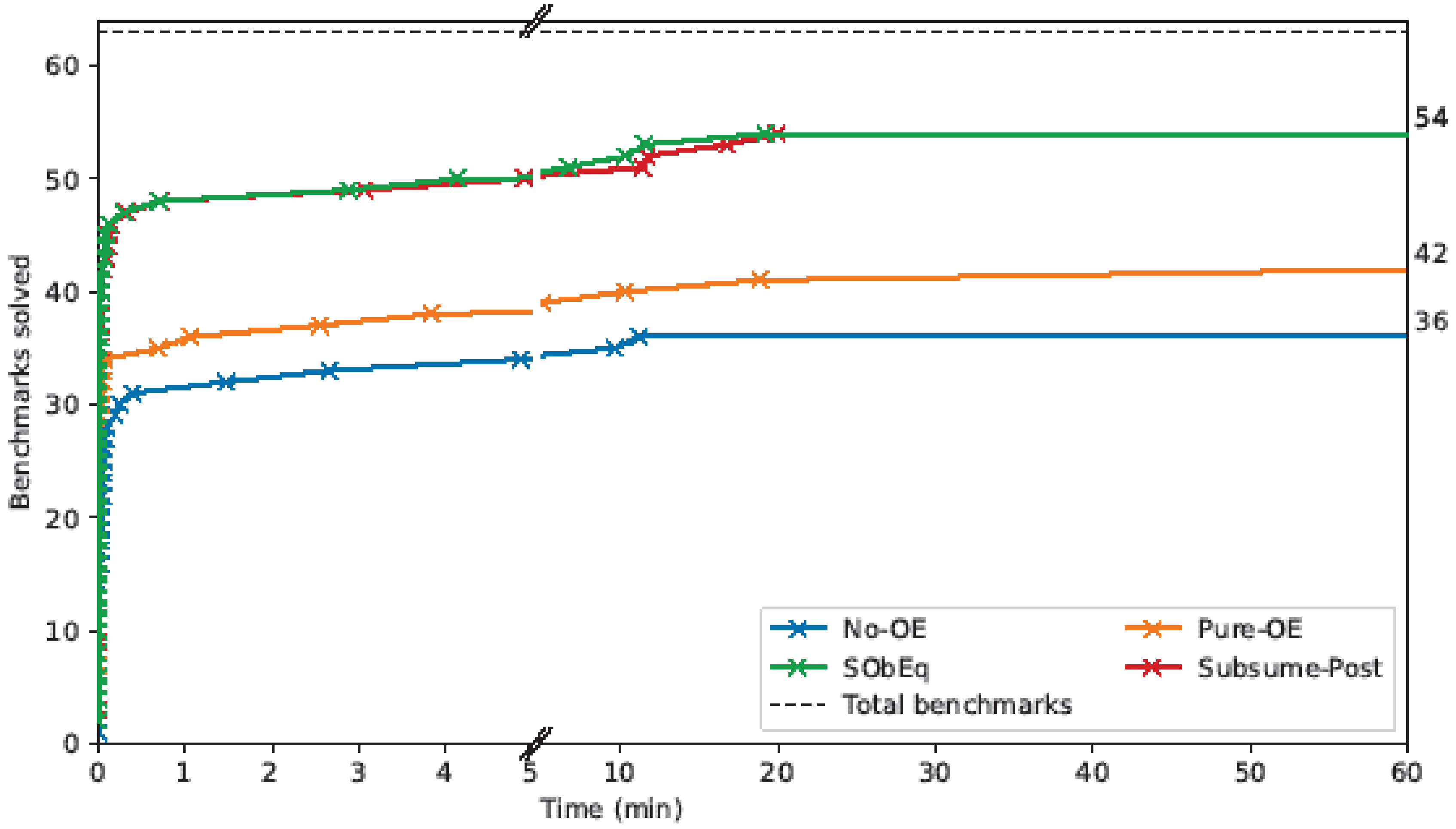
$\{\texttt{n} \mapsto 5\}\ \texttt{n + 1}\ \{\texttt{n} \mapsto 5; 6\}$

$$\dfrac{\{\texttt{n} \mapsto 5\}\ \texttt{n}\ \{\texttt{n} \mapsto 5; 5\} \qquad \dfrac{\{emp\}\ \texttt{0}\ \{emp; 0\}}{\{\texttt{n} \mapsto 5\}\ \texttt{0}\ \{\texttt{n} \mapsto 5; 0\}}\ \textsc{Frame}}{\{\texttt{n} \mapsto 5\}\ \texttt{n + 0}\ \{\texttt{n} \mapsto 5; 5\}}\ \textsc{Eval}$$

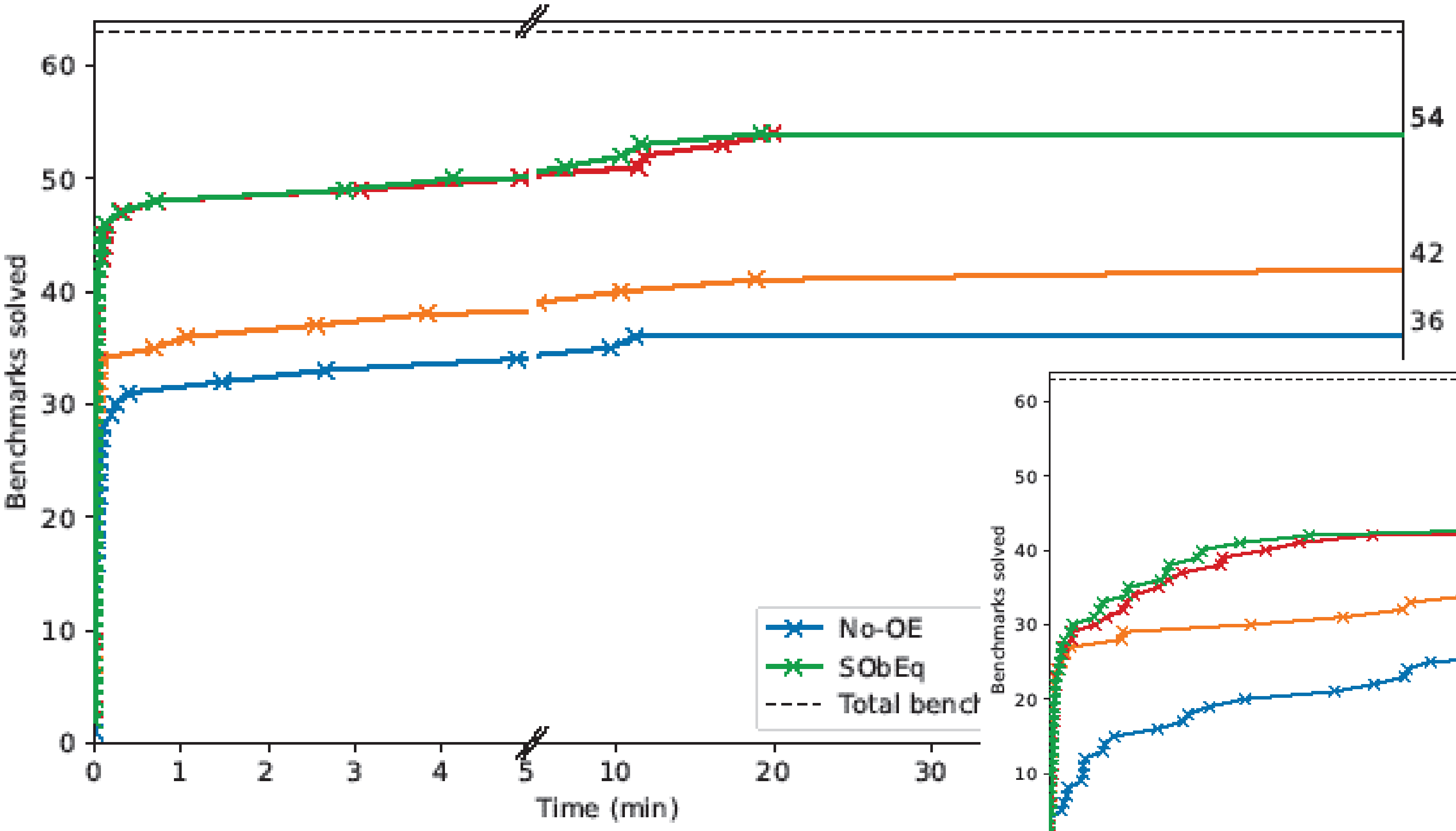
Results: pruned programs

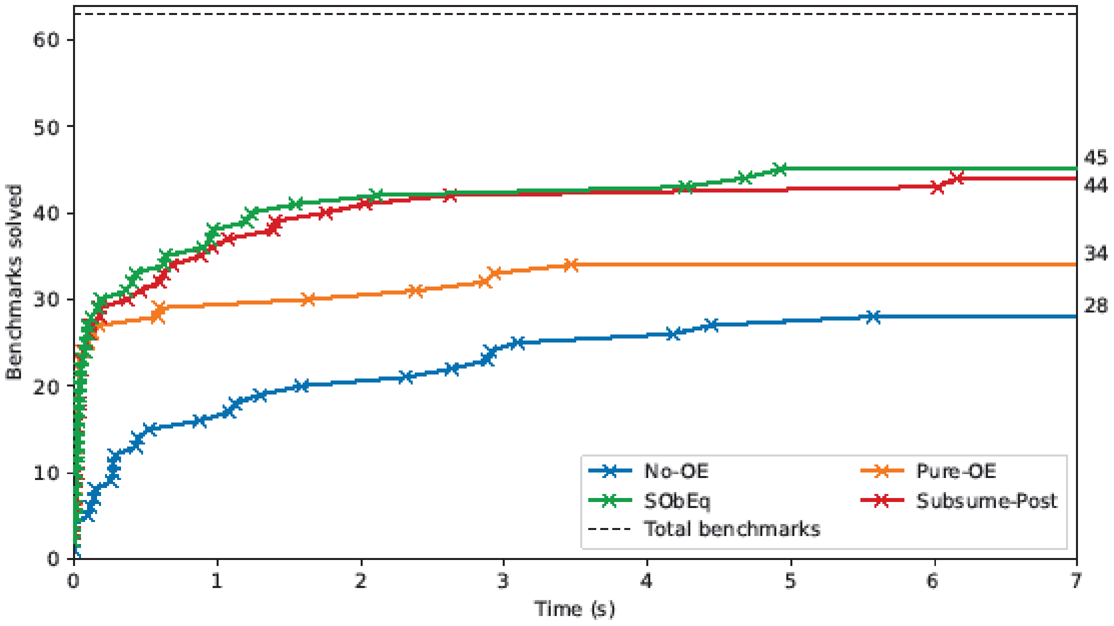


Results: Pruning gives us speed



Results: Pruning gives us speed





Results: determinism and quality

FrAngel: Shi et al. 2019

```
arr.splice(beg++,1,toadd).reverse();
return end + 1;
```

Results: determinism and quality

FrAngel: Shi et al. 2019

```
arr.splice(beg++,1,toadd).reverse();
return end + 1;
```

return temp array

Results: determinism and quality

FrAngel: Shi et al. 2019

```
arr.splice(beg++,1,toadd).reverse();
return end + 1;
```

return temp array

reverse & discard

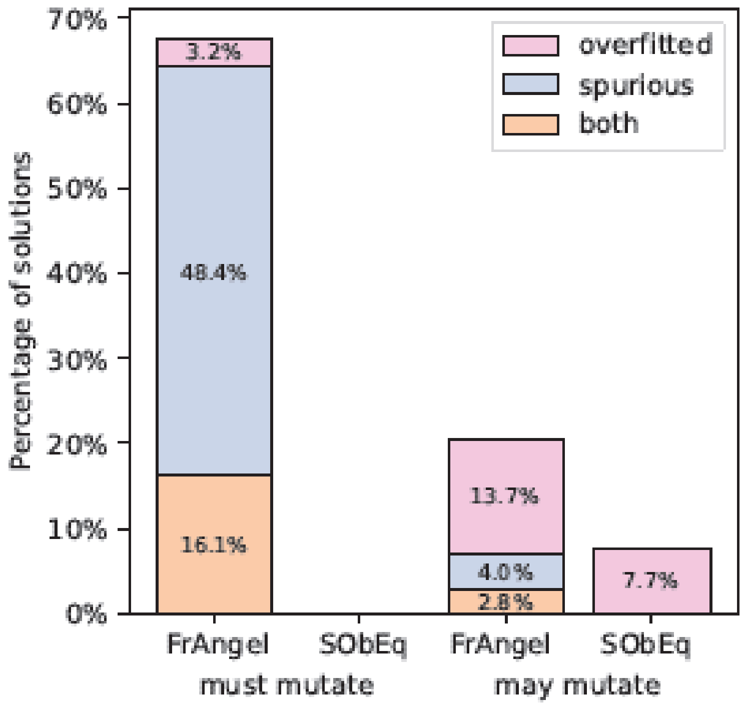
Results: determinism and quality

FrAngel: Shi et al. 2019

```
arr.splice(beg++,1,toadd).reverse();
return end + 1;
```

return temp array

reverse & discard



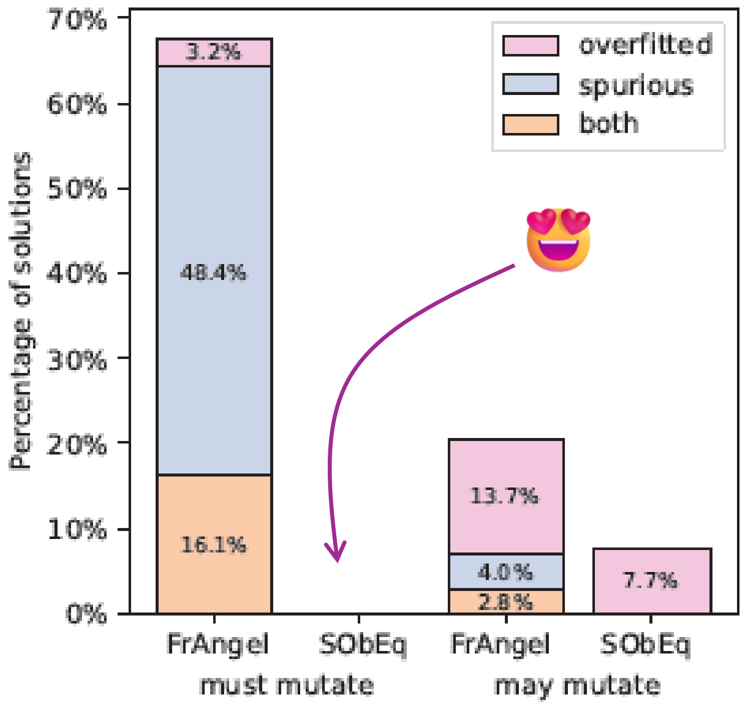
Results: determinism and quality

FRANGEL: Shi et al. 2019

```
arr.splice(beg++,1,toadd).reverse();
return end + 1;
```

return temp array

reverse & discard



***S**ide-effects in **OB**servational **EQ**uivalence*

1) Change representation to triples

$$\underbrace{\{x \mapsto v_x * \ y \mapsto v_y, \dots\}}_{\text{precondition}}\ p\ \underbrace{\{x \mapsto v_x' * y \mapsto v_y', \dots ; r\}}_{\text{postcondition}}$$

precondition

postcondition

result

2) Combine them using Separation Logic

$$\frac{\{P_1\}\, p_1\, \{P_2; r_1\} \quad \{P_2\}\, p_2\, \{P_3; r_2\} \quad \cdots \quad \{P_k\}\, p_k\, \{P_{k+1}; r_k\} \qquad (c(r_1, \dots, r_k), P_{k+1}) \to (r, Q)}{\{P_1\}\, c(p_1, \dots, p_k)\, \{Q; r\}}\ \textsc{Eval}$$

$$\frac{\{P\}\, p\, \{Q; r\}}{\{P * R\}\, p\, \{Q * R; r\}}\ \textsc{Frame}$$

Result: correct enumeration with mutations!