Abstraction-Based Interaction Model for Synthesis

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Programming by Example

Task: find the most frequent bigram in a string.

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"bd"
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`input.substring(1,3)`

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Programming Not Only by Example

Input
⇓
Output

Peleg, H., Shoham, S., & Yahav, E. (2017). Programming Not Only by Example. *ICSE 2018 (Upcoming)*.

Programming Not Only by Example

$$\{m \in M \mid [\![m]\!](input) = output\}$$

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$\{m \in M \mid \textcolor{red}{\boldsymbol{p}}(m)\}$

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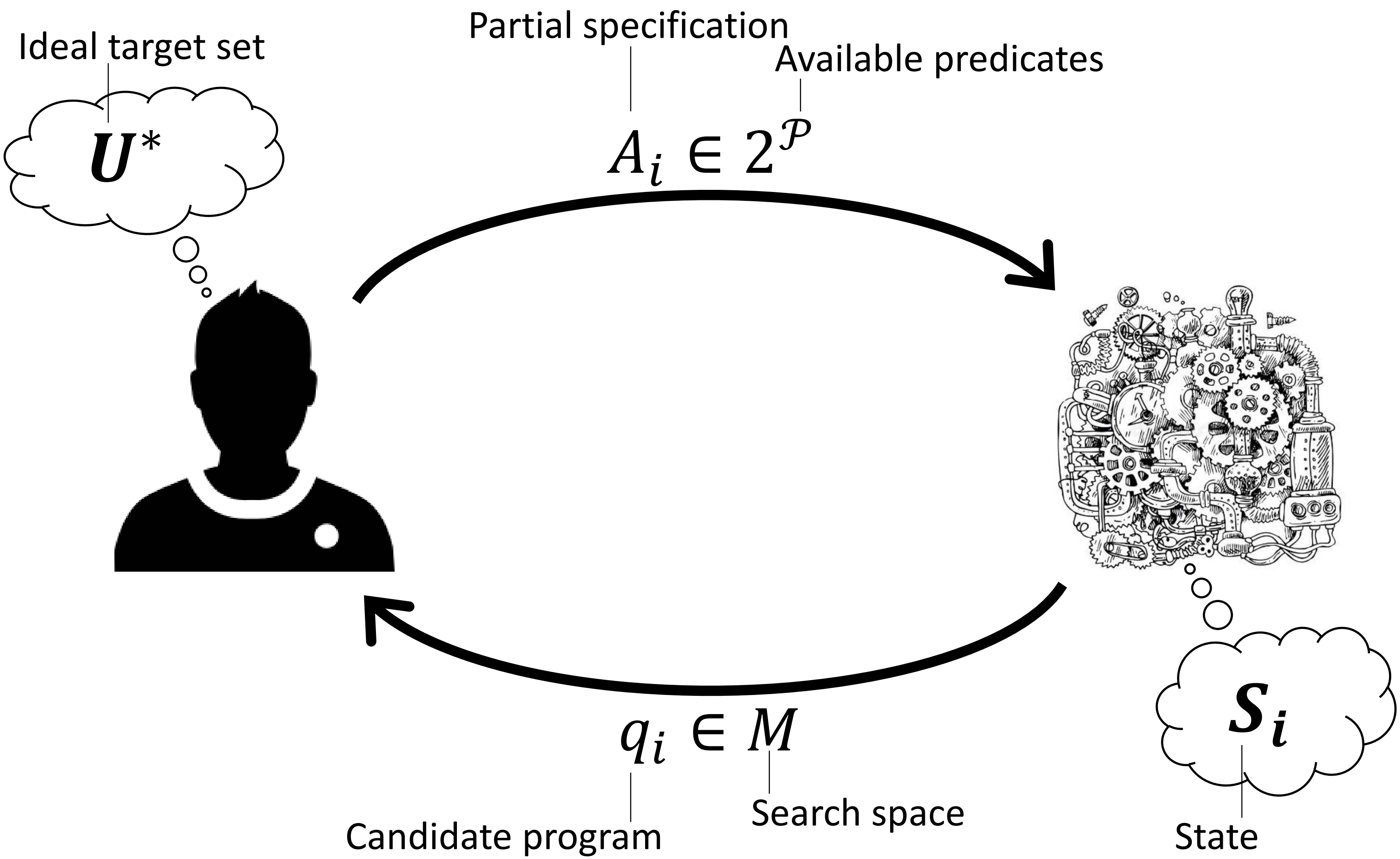
Our Goal

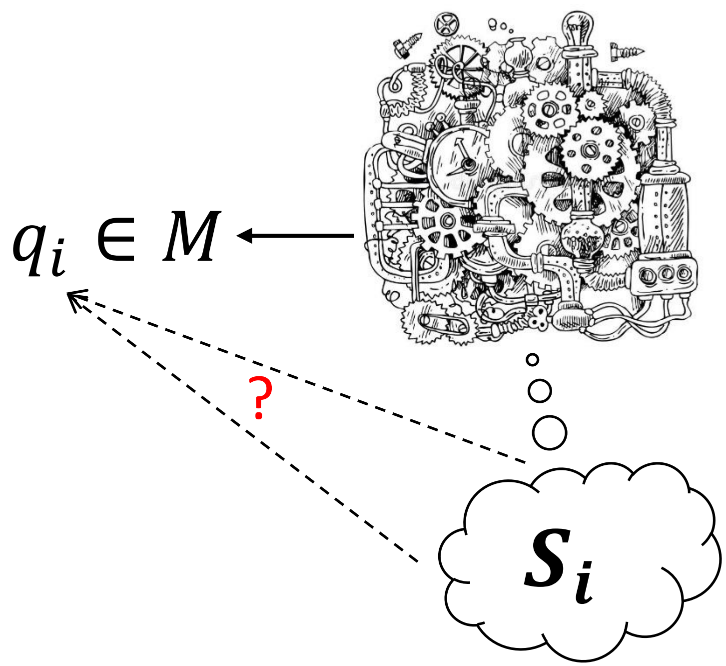
- To model user-driven synthesis
  - Works in practice but we do not understand its limitations
- Properties
  - Of the synthesizer
  - Of the user
- Guarantees
  - Termination (in paper)
  - Are “bad sessions” recoverable

Iterative, interactive synthesis



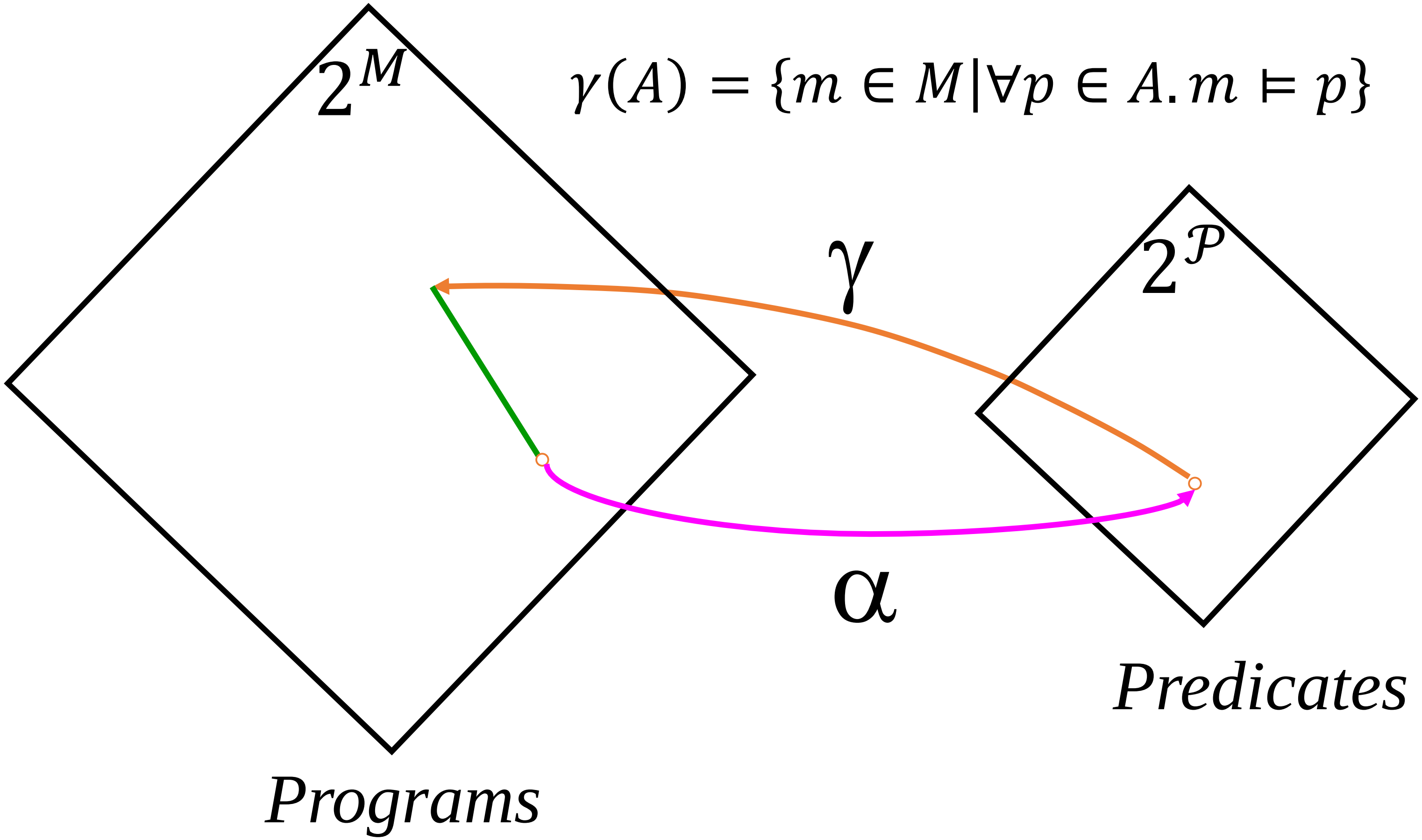
Select

- Candidate program is selected via *some* selection criterion: $Select$
- $Select$ usually designed to return a program from U^* ASAP (in 1-2 iterations)
- There is little theoretical work about *the long run*



An abstract domain

$$\gamma(A) = \{m \in M | \forall p \in A.\, m \vDash p\}$$



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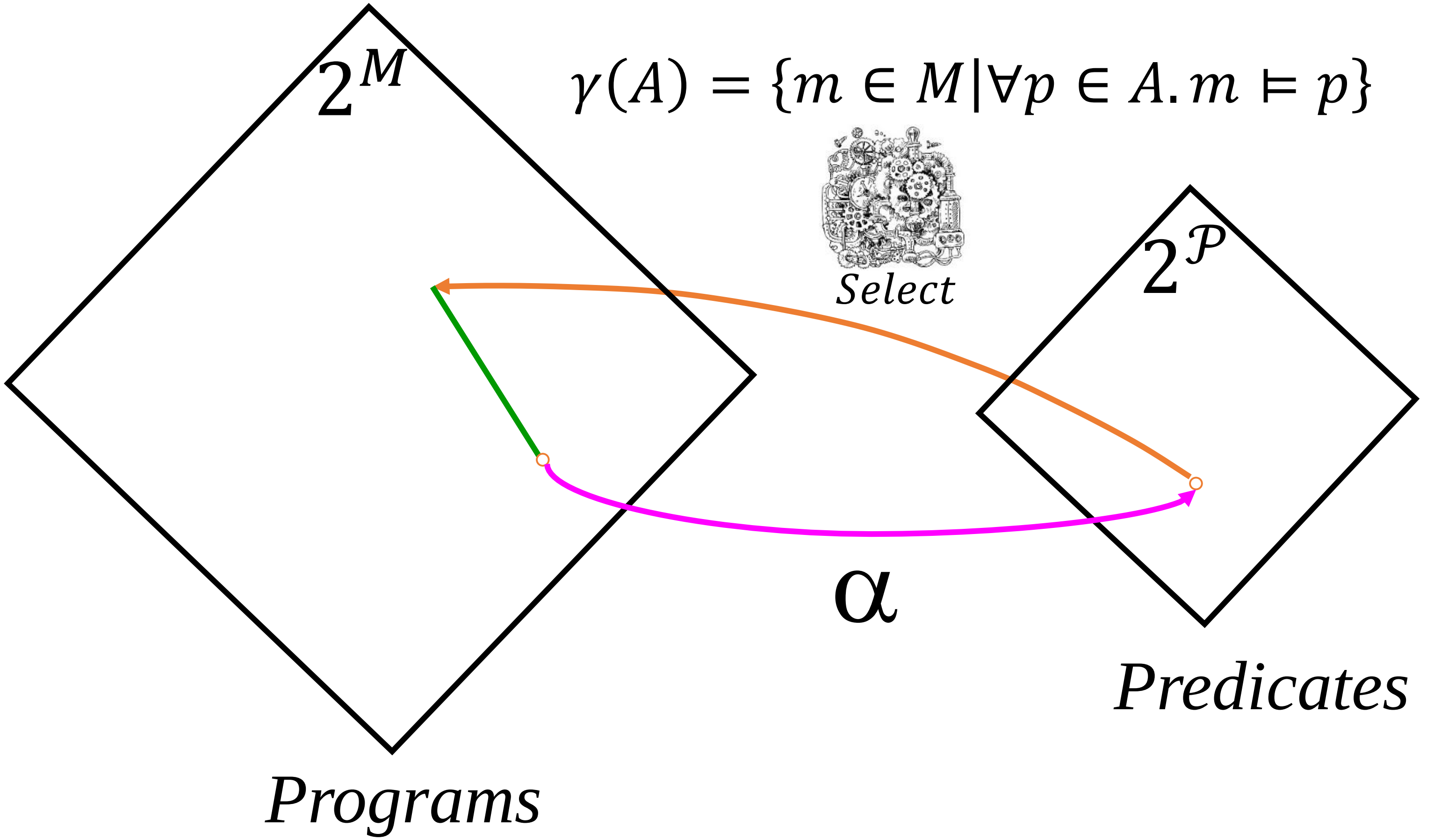

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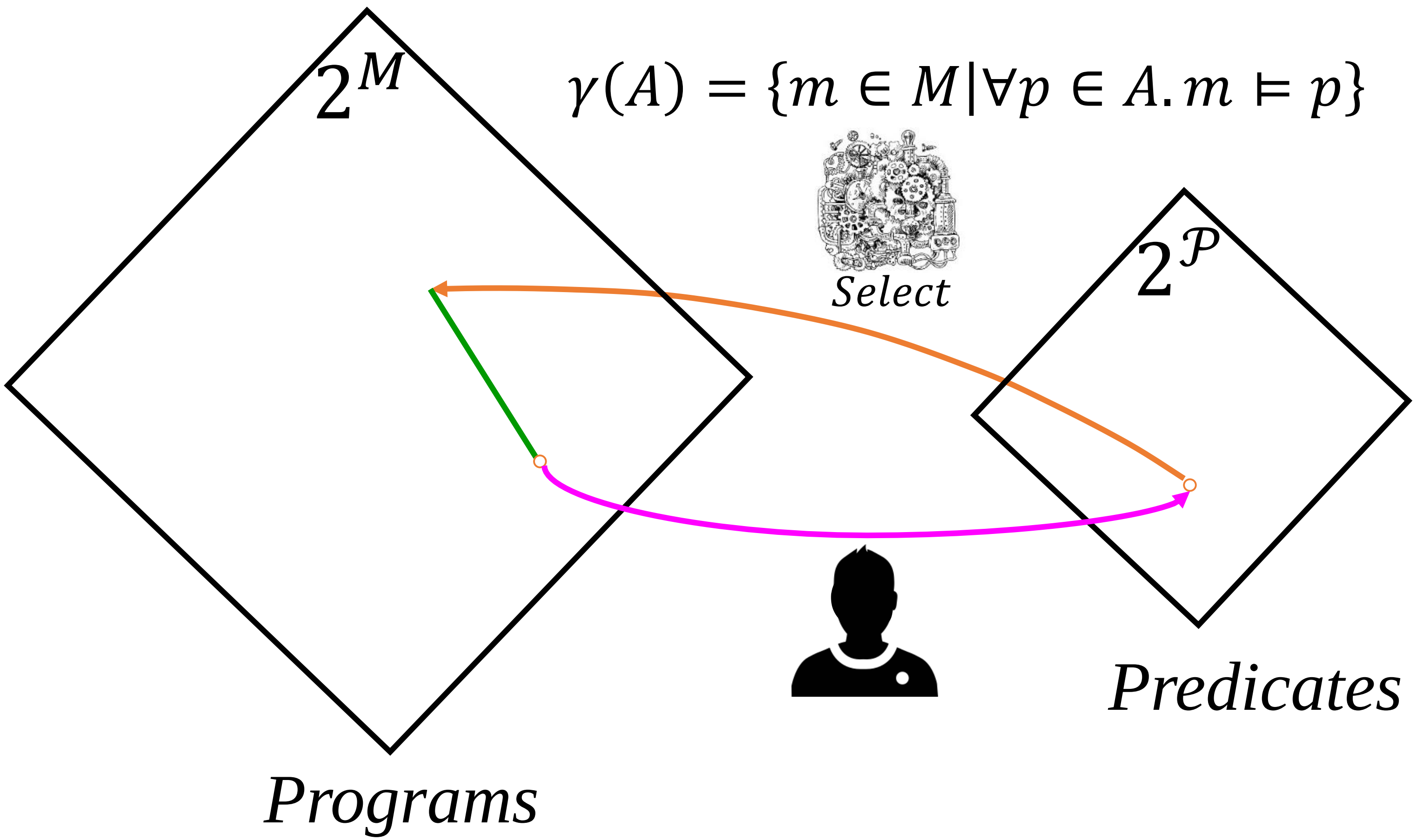

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Synthesis Session

- A synthesis session:

Candidate program

$$\mathcal{S} = (A_0, q_1)(A_1, q_2)\ \dots$$

Initial specifications

User answer

  - Synthesizer state: $S_i = S_{i-1} \sqcap A_i$
  - $q_i = Select(S_{i-1})$, or $q_i \in \gamma(S_{i-1}) \cup \{\perp\}$
  - If $\mathrm{q_i} \in U^* \cup \{\perp\}$, the session terminates

Synthesis user

- The user is aiming for some *ideal* set of programs $U^* \subseteq U$ (where U is the universe of all programs)
- The realizable target set is $M^* = M \cap U^*$
- **Correctness:** A user step is correct when

$$A_i \subseteq \{p \in \mathcal{P} \mid \exists m \in U^*. m \vDash p\}$$

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$\boldsymbol{U^*}$

```
(1 until n).fold((x,z)=>x+z)
```

```
sum(range(1,n+1))
```

```
var sum=0
for(int i = 1; i <=n; ++i) sum += i
return sum
```

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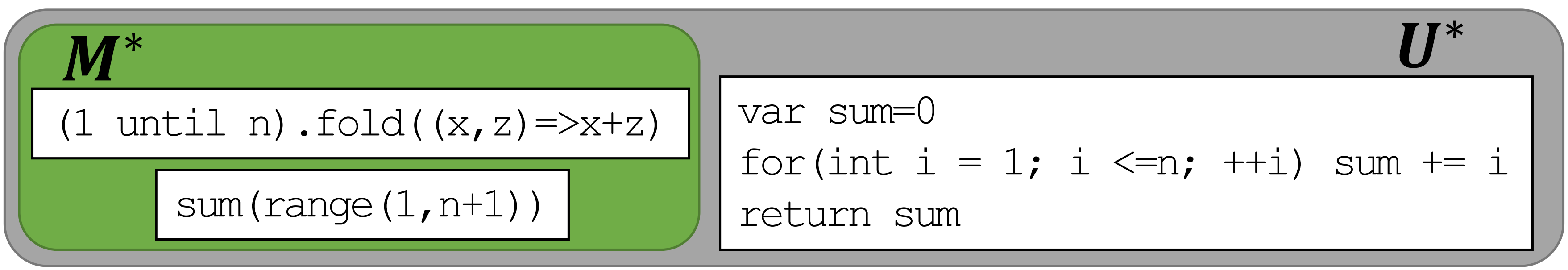

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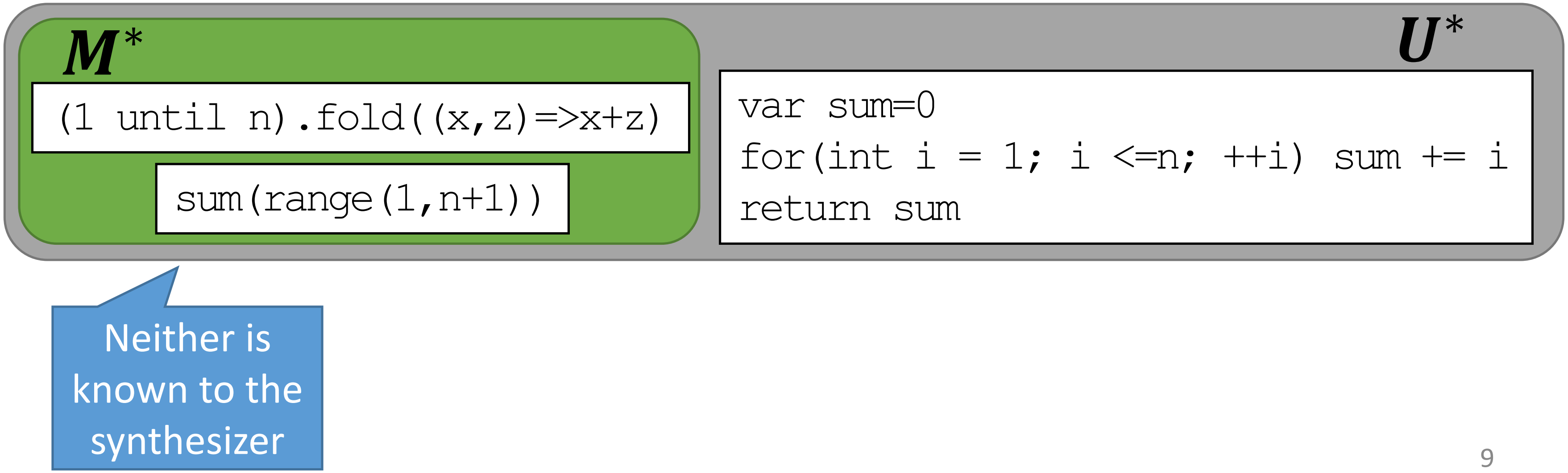

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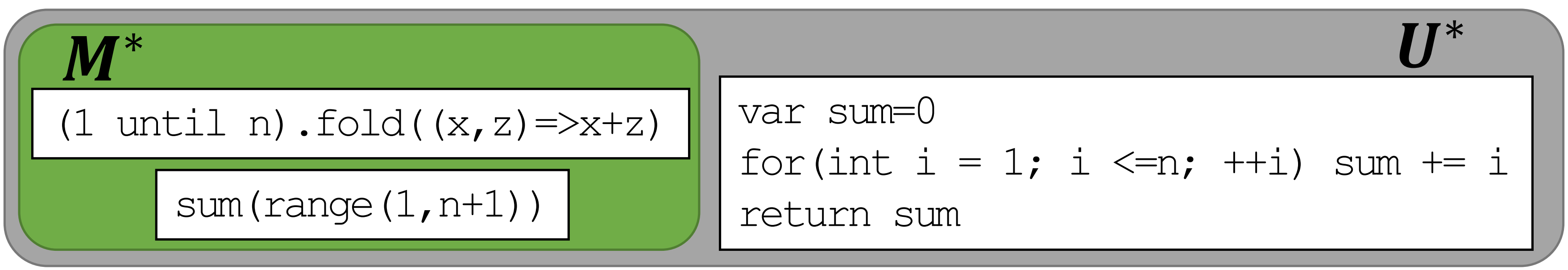

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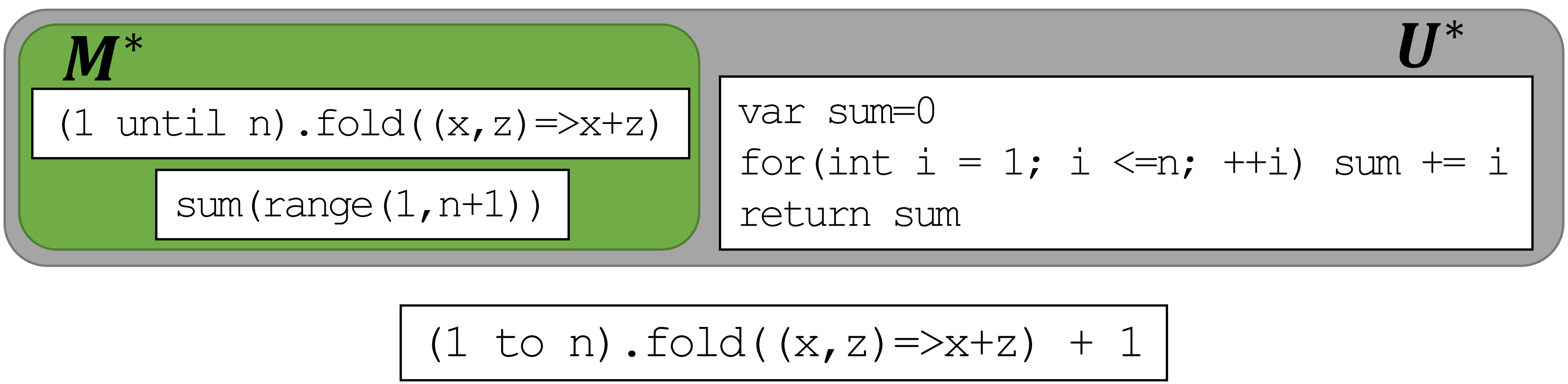

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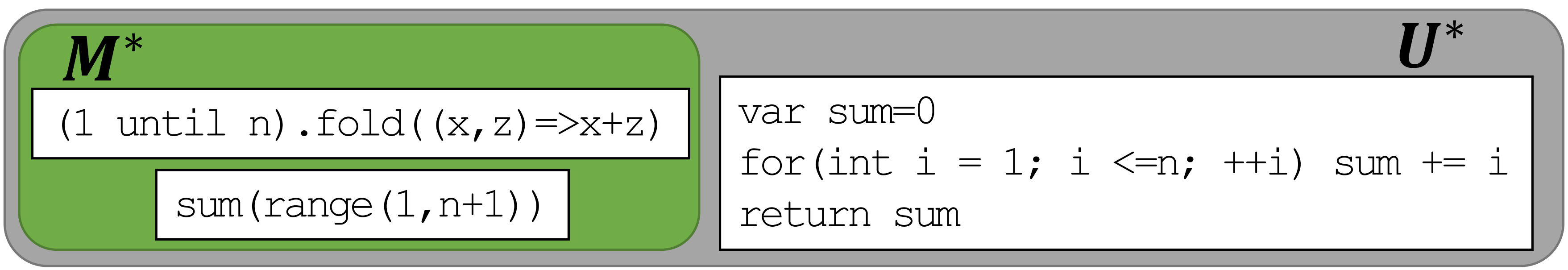



```
(1 to n).fold((x,z)=>x+z) + 1
```

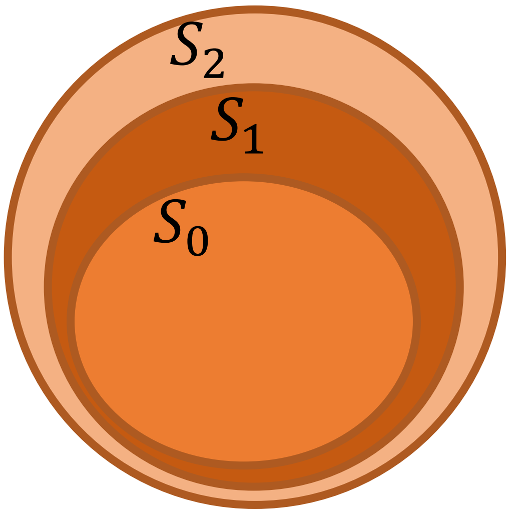
✓ ✗

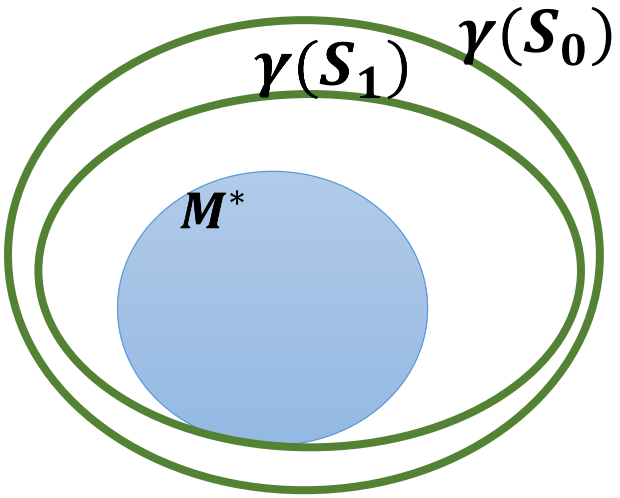
User guarantees

- In a synthesis session:
  1. The user is correct for as long as possible. When not possible, $A_i = \perp$
  2. The user will always accept a program from M^*

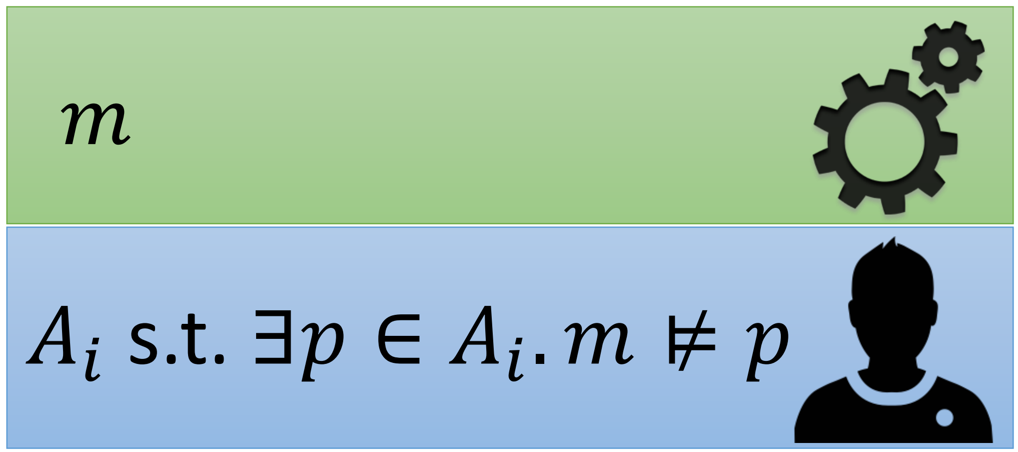
Progress

- Adding new predicate doesn't guarantee that the session is progressing
- $S_{n-1} \sqsubset S_n$ could still mean that $\gamma(S_n) = \gamma(S_{n-1})$
- Synthesizers usually don't check





Easiest way to make progress



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A_i s.t. $\exists p \in A_i . m \nvDash p$

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- Rule out *at least* the current program

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- In other words: something that's wrong with the current program
- Rule out *at least* the current program
- Important when $q_i \in \gamma(S_{i+1}) \Rightarrow Select(S_{i+1}) = q_i$
- Easy to check, but too strong

A Different Model of Progress

- A_i makes *weak progress* if
  $$\gamma(S_{n-1} \sqcap A_i) = \gamma(S_n) \subsetneq \gamma(S_{n-1})$$
- We can provide positive reinforcement

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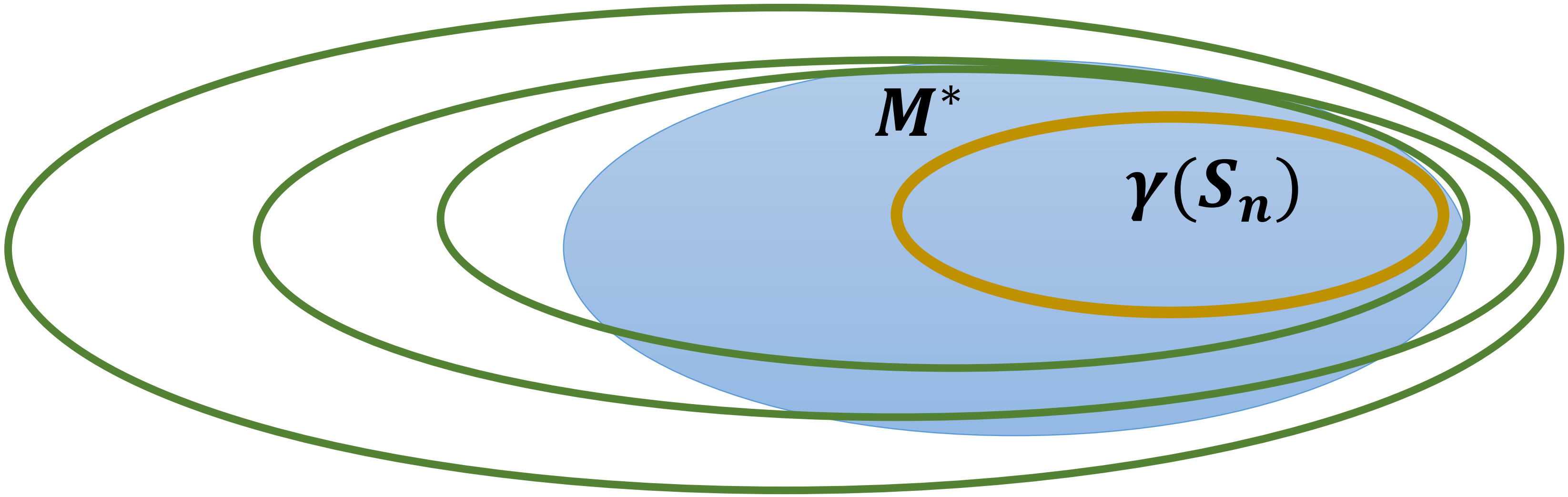
✓

- Harder to check: $S_i \nRightarrow A_i$
- Once we know there is progress, we know some things about **termination** (see paper)

Convergence

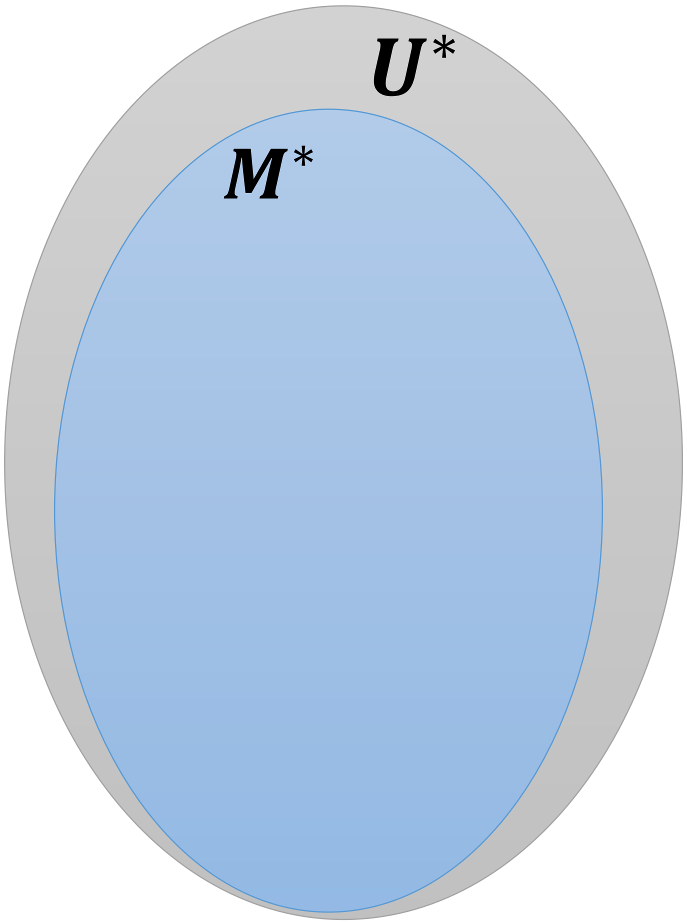
- A session **converges** if $\gamma(S_n) \subseteq M^*$
  - User correctness means the session ends at state n
- **Converges successfully**: $\emptyset \neq \gamma(S_n) \subseteq M^*$



Core set

- Core set: the set of **finite** underapproximations of M^*

$$\mathcal{B} = \left\{ B \subseteq \mathcal{P} \,\middle|\, \begin{array}{c} 0 \neq \gamma(B) \subseteq M^* \\ \wedge\, |B| \in \mathbb{N} \end{array} \right\}$$



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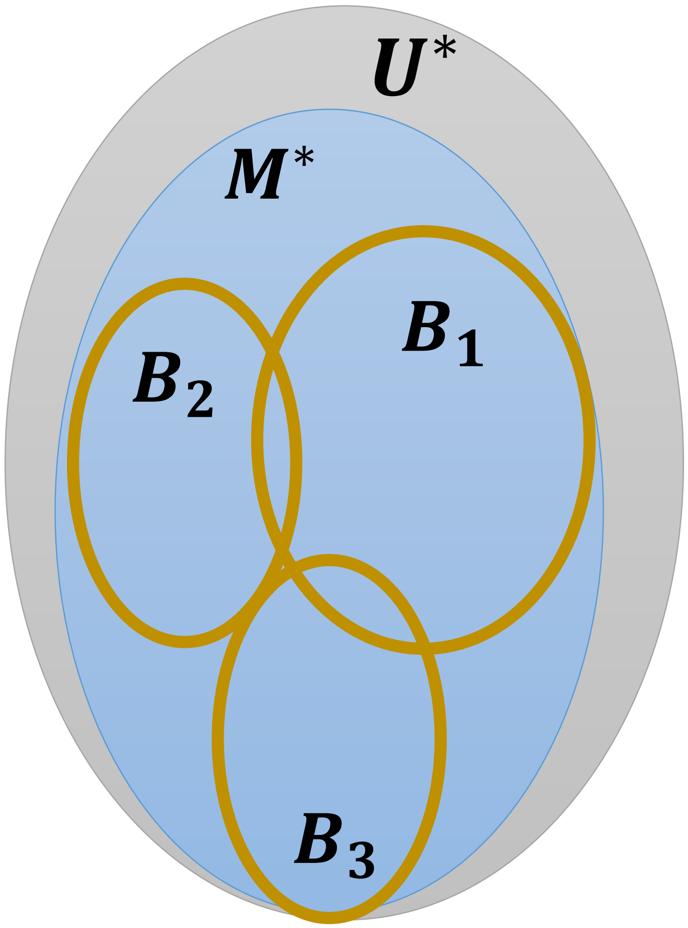

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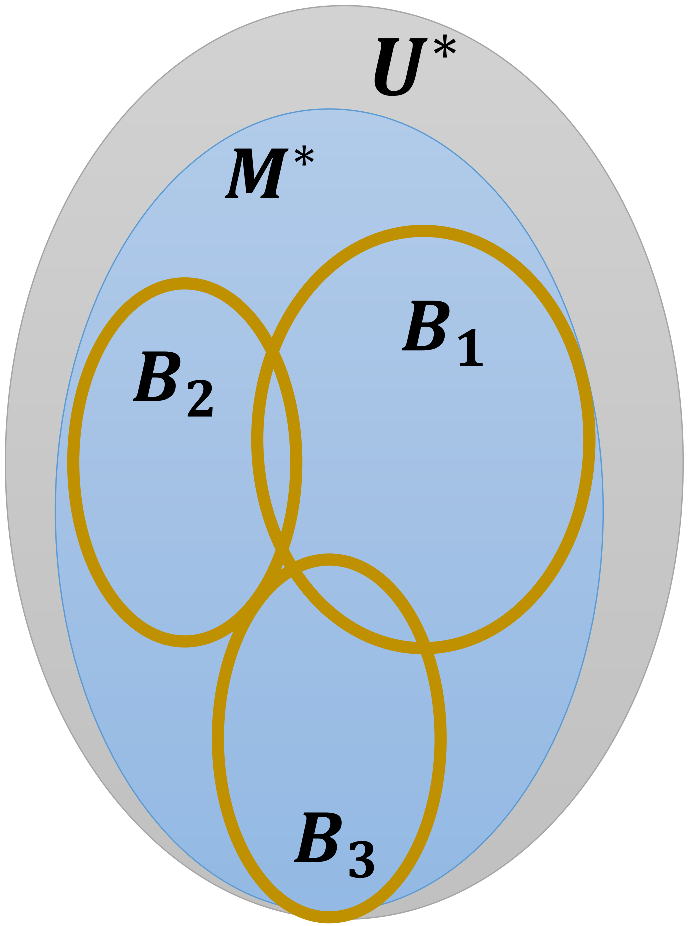
- User intention is realizable if $M^* \neq \emptyset$
- $\boldsymbol{\mathcal{P}}$**-realizability**: can converge under $\mathcal{P}$



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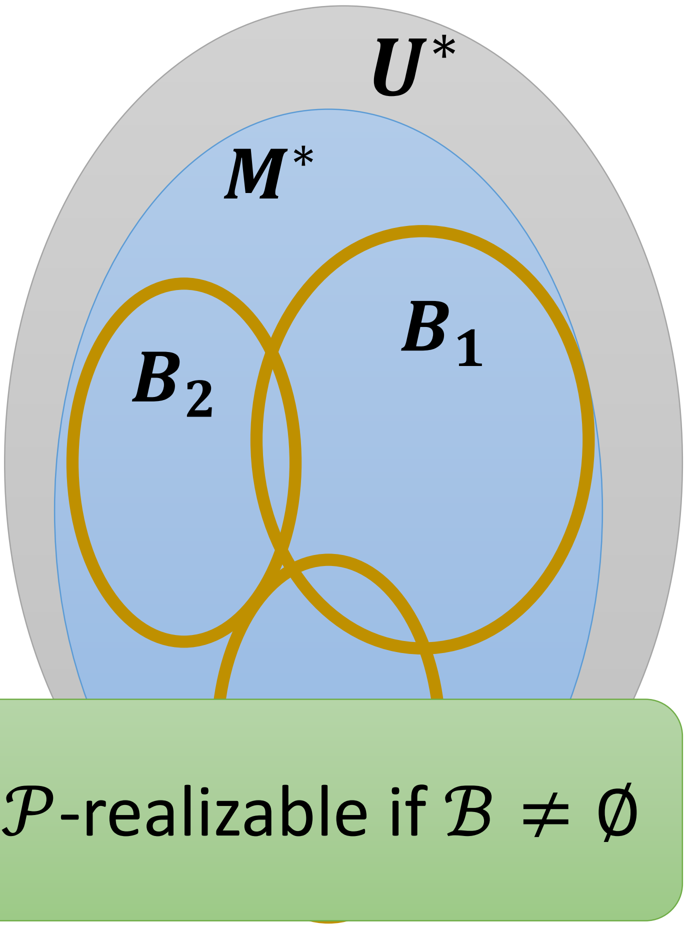
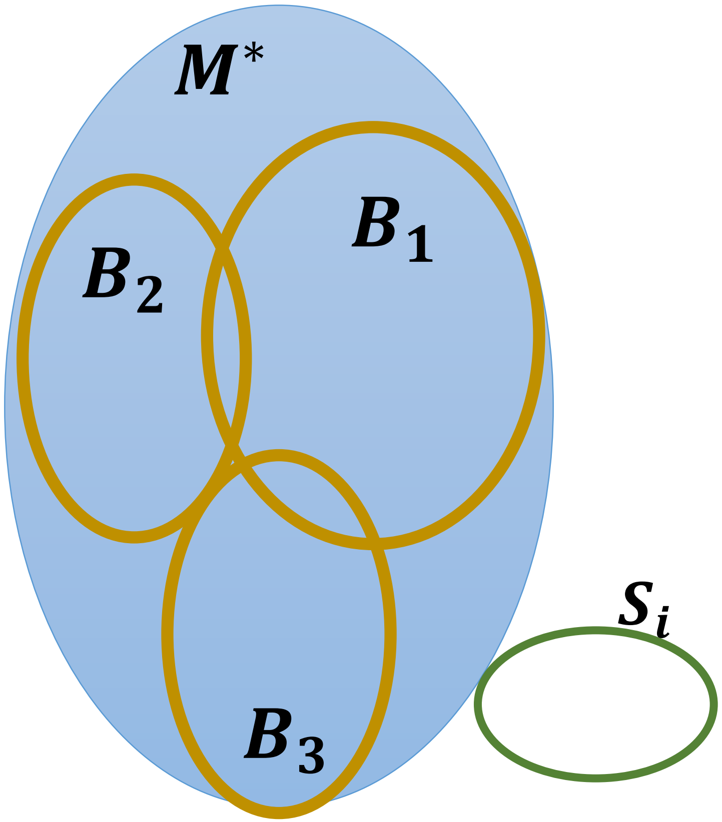


M^* is $\mathcal{P}$-realizable if $\mathcal{B} \neq \emptyset$

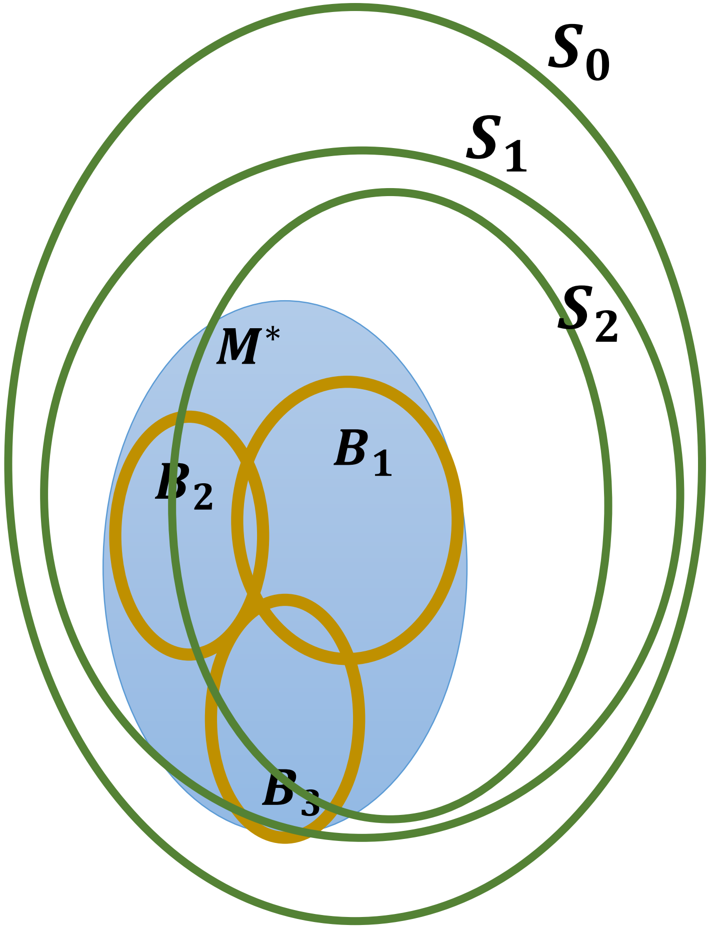
Infeasible point

- A state where
$$\gamma(S_i) \cap M^* = \emptyset$$
- $Select$ can't succeed, even in best case
- The **First Infeasible Point** is the first point of **failure**
- Can we **backtrack** before an infeasible point?



Point of inevitable failure

- State S_i is an POIF if $\forall B \in \mathcal{B}. \gamma(S_i) \cap \gamma(B) = \emptyset$
- Specifically, S_i is an POIF if $\gamma(S_i) \cap M^* = \emptyset$



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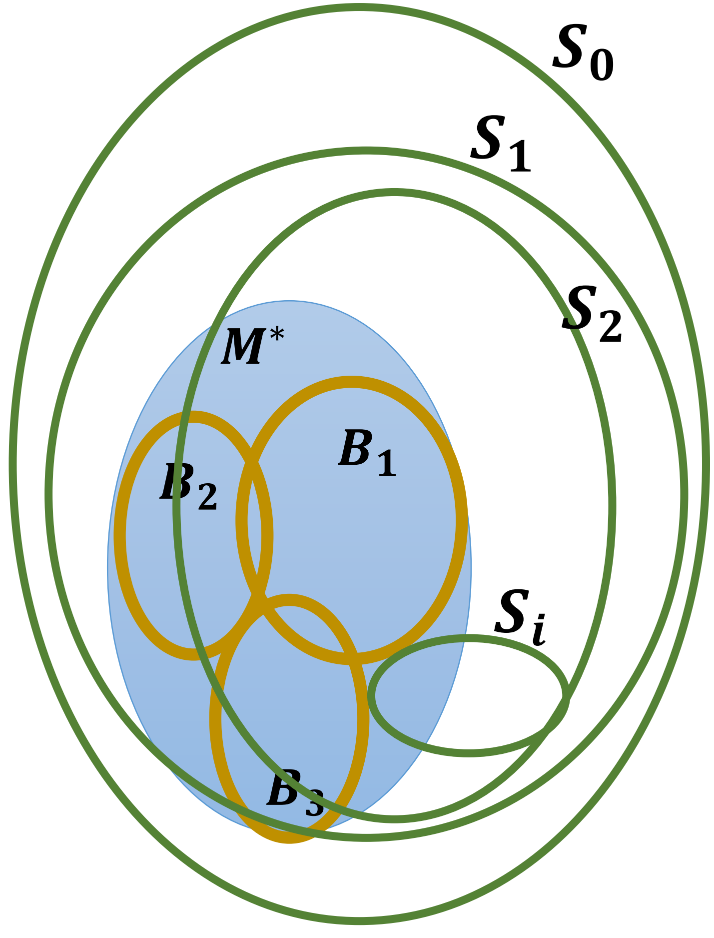

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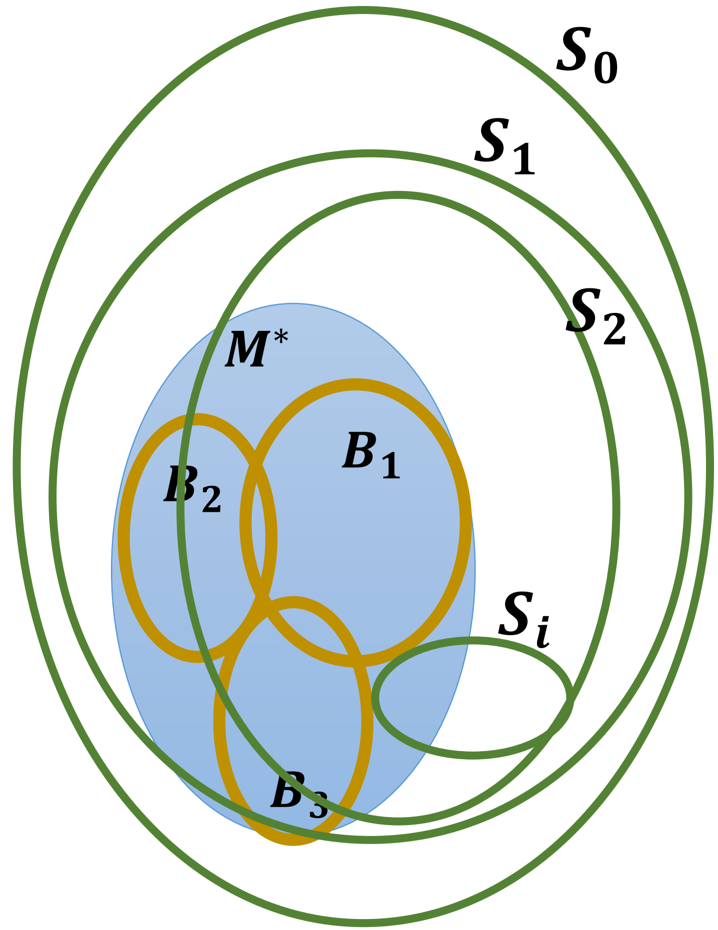


- As long as S_i is not an POIF we can still converge
- Can we **backtrack** before an inevitable point of failure?

Backtracking from failure

Theorem: for any $k \in \mathbb{N}$ there exists a session $\mathcal{S}$ where state S_i is an point of inevitable failure and only state S_{k+i} is the first infeasible point.

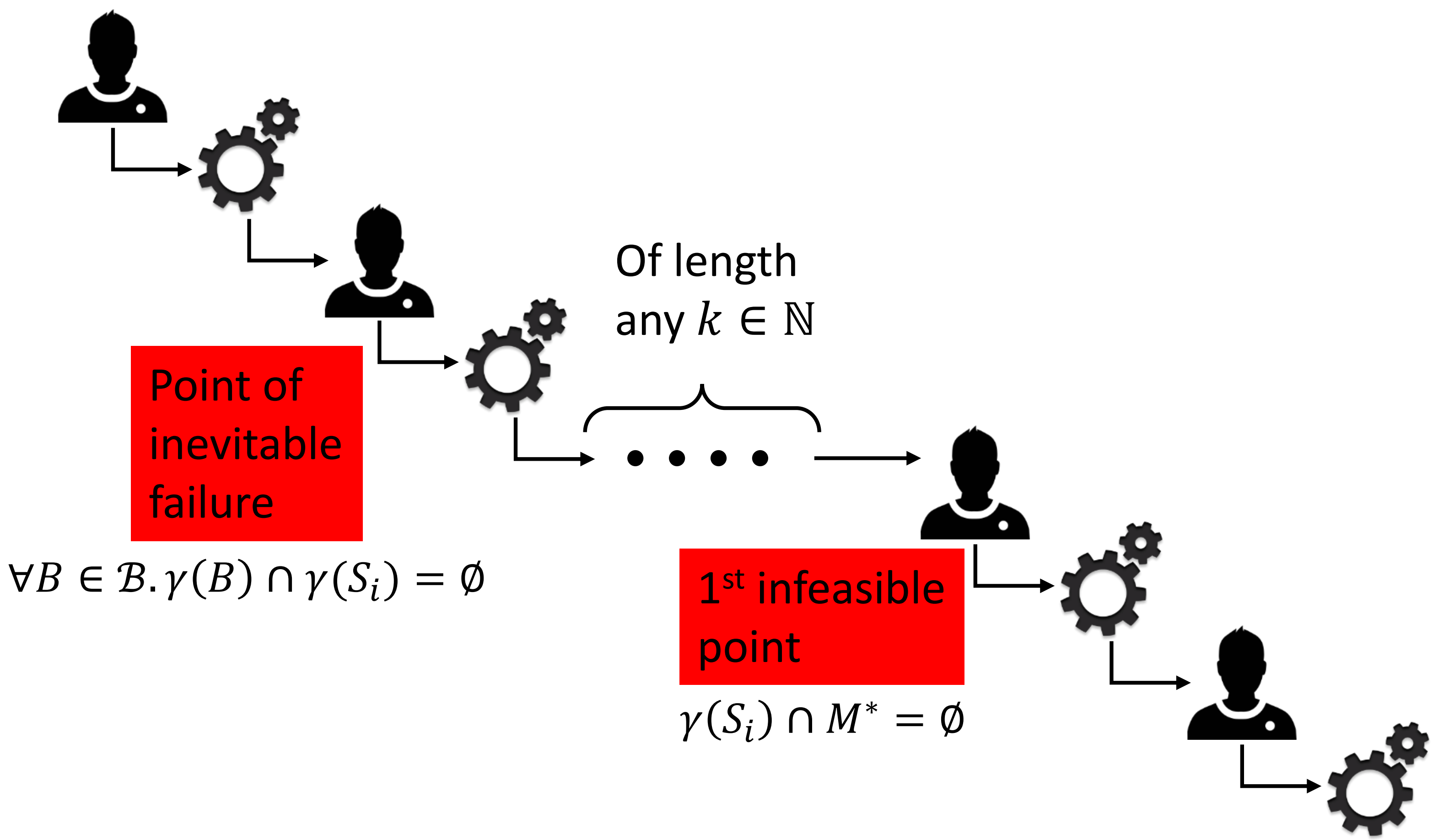
Backtracking from failure

Theorem: for any $k \in \mathbb{N}$ there exists a session $\mathcal{S}$ where state S_i is an point of inevitable failure and only state S_{k+i} is the first infeasible point.

Essentially: there is no bound on the number of steps to backtrack once failure is apparent.

Proof: by construction

An unbounded session



Construction

- Candidate program space M is spanned by:
  - if-else
  - ==
  - lists of ints ([],[1,2], etc.)
  - recursive call `f`
  - the input variable `i`
  - cons
  - max
  - remove
  - sort
  - reverse

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Example candidate:

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else cons(max(i),
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- Available predicates in $\mathcal{P}$:
  - Input-output examples
  - $exclude(e)$, for any program element e

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exclude(==)

$\perp$

Conclusion

- An iterative, interactive model of synthesis
- An abstract domain of predicates
- Progress
- Convergence
- The unboundedness of backtracking
- We hope these results help future synthesizer designers